Bank of Israel **Research Department**

The State-of-The-Economy Index and The probability of Recession: The Markov Regime-Switching Model[1]

by

Arie Marom*, Yigal Menashe* and Tanya Suchoy*

Discussion Paper No. 2003.05
June 2003

---

[1] The paper was presented at the 19-th annual conference of the IEA (2003, June). The authors thank Simcha Bar-Eliezer, Yossi Gibre and Akiva Offenbacher for their helpful remarks, and Kfir Kalish for his excellent assistance in processing the data and performing the statistical tests.

* Bank of Israel, Research Department. http://www.boi.gov.il

מחלקת המחקר, בנק ישראל ת״ד 780, ירושלים 91007
Research Department, Bank of Israel, POB 780, Jerusalem 91007, Israel

Abstract

This paper offers a new approach to the identification of the business cycle in Israel, using a composite state-of-the-economy index based on the Markov regime-switching model. This model, unlike its predecessor, is not bound by the assumption of symmetry between periods of recession and expansion. The composite index is estimated simultaneously with the probability of the economy being in a recession. Several improvements have been incorporated into the index components: the series of goods exports was included, an appropriate step in the light of the fact that Israel is a small open economy; the lag in the connection between employment and economic activity was also dealt with. The new composite index presents a sharper picture of the business cycle than did the old Bank of Israel index. The new index, with the probability of recession, give an accurate description of the business cycle *ex post facto,* with regard to their high correlation with the GDP, final uses and employment series, and with regard to identifying the turning points in the cycle in real time. The probability of recession successfully detects a downturn immediately, and more sharply about two months after its onset. The start of expansion, however, takes longer to detect, as occurred with the period of growth in high-tech that began in the second half of 1999. Another advantage of the new index, and particularly of the recession probability, over the old index lies in their being less volatile and less liable to retroactive revisions.

1. Introduction

It is important to policymakers and to the business sector to qualify economic situation and to recognize turning points in business activity. Intuitively the business cycle - expansion and recession – can be identified by one central variable such as GDP, but the cumulative experience gained in empirical analyses shows that it is preferable to identify business cycles via a composite index comprising a group of indicators that represent different aspects of economic activity. As a rule, the GDP update is released after the main monthly statistics, such as industrial production, employment and trade have been published. Another GDP "failure", from the point of view of current monitoring, relates to its low (quarterly) frequency. Finally, like any single series, it contains random noise – especially at the end of the series – that can be separated only by looking for common pattern dynamics of group of indicators.

Since the early 1990s, many countries throughout the world have adopted composite indices as cyclical indicators.[1] They almost all include as components the labor market indicators, e.g., the number of employed persons, wages, and unemployment; series that embody supply-side factors, such as the index of manufacturing production; and variables that reflect supply and demand and that are correlated with consumption, such as disposable income and indices of sales and revenue. In some countries the indicators also include trade indices, represented by series of the value of imports and exports. In Israel the composite index comprises the following monthly series: the index of manufacturing production; imports, excluding capital goods; trade and services revenue; and the number of business-sector employee posts. A fifth component added to the index in the new model is goods exports excluding agriculture, fuel, diamonds, ships and aircraft.

In this study we exploit a concept of "growth cycles", rather than "business cycles"[2]. Unlikely the classical business cycle definition, which differs between periods of absolute contraction (with negative growth rates) and expansion, the growth cycle differs the rapid economic growth, on the one hand, and slowdowns in activity, on the other , when the rates of growth may remain positive but lie below their long-run average.

The proposed model enables currently compiling of the composite index, simultaneously with the probability of recession. This additional measure enriches the picture, obtained from the composite index, and ensures diagnostics, being more monotonous.

The paper is organized as follow. Section 2 contains a brief review of the main methods used to describe the structure of the business cycle and to select appropriate index components. Section 3 introduces the Markov regime-switching model, which provides the new index and the probability of recession for Israel’s economy. Section 4 analyzes the estimation results and especially the *ex post facto* explanatory power of the new index. The fifth section focuses on the new index’s ability to recognize the economic situation in real time according to the information, known at a given time. Section 6 concludes.

---

[1] For the review we refer to the Conference Board site : http://www.tcb-indicators.org
For details of the weights of the components of the coincident indices in different countries see Appendix A.

[2] For the whole discussion on these two concepts see Zarnowitz and Ozyildirim (2002).

2. Measuring the business cycle in Israel and worldwide

The first empirical studies analyzing the business cycle were those of Burns and Mitchell (1946), who examined a very broad range of series of variables relating to the US economy, in an attempt to discover some common feature that would describe the business cycle. The macroeconomic variables were divided into three categories: lagged, coincident or leading indicators of economic activity.[3] Despite their considerable efforts, the researchers did not find indicators that would successfully identify the business cycle, and in particular, that would identify its turning points in real time. This was mainly due to difficulties related to the lack of utilization of all the data required to characterize the business cycle and insufficient formal analysis.

Another direction of inquiry is the connection between the business cycles of different economies. Moore (1983) and Boehm and Moore (1984) studied the import and export of business cycles via financing and trade channels. Their empirical findings led them to include trade-related series among the components of the indicators of the state of economic activity.

A major stride forward in the field of composite indices was made in the 1990s, in the studies by Stock and Watson (1989). They developed a linear stochastic model, in which the common fluctuations of coincident economic indicators have been explained by an unobservable dynamic factor, estimated recursively, through the relative changes in these variables from their previous state (State Space Model). This approach was adopted by many countries in the first half of the 1990s.

With the development of econometric tools, there were notable attempts to generalize the common experience of composite indices construction, like Boehm (2001). In particular, he summarizes the most important requirements of the components of the coincident index as follows: these indicators should cover the main areas of the business activity and represent demand-supply sides of the principal industries, the labor market and the balance of payments. They must cause a minimal number of false alarms regarding turning points in the business activity; must be available at high (at least monthly) frequency, as soon as possible after the end of the period to which they relate; and finally, they should not be subject to large retroactive revisions.

Although the weighted linear indices became widely adopted, the diagnostic power of it may be contested until the remarkable drawback would be overcame: the linearity imposed by the estimation process creates symmetry between periods of economic contraction and periods of growth of activity, and hence makes the periods similar in their intensity and length. This fact becomes especially problematic in the light of the empirical findings of studies of the business cycle worldwide and specifically in the US, which show that periods of growth are more gradual and of longer duration than periods of contraction of activity.

Hamilton (1989) modeled the business cycle asymmetry by assuming that the quarterly rate of GNP growth followed stochastic nonlinear process, which could switch between

---

[3] As opposed to leading indicators, coincident indicators are not expected to forecast economic activity accurately.

the two regimes (expansion and recession), in accordance to the Markov chain transition probabilities. Hamilton's results supported the asymmetric concept; however, the model did not pinpoint the recessions in the US as classified by the NBER. One explanation of this problem may be the use of single GNP series, instead of composite index, which lacked some more information to characterize the business cycle, in addition to which it also suffered from notable noise.

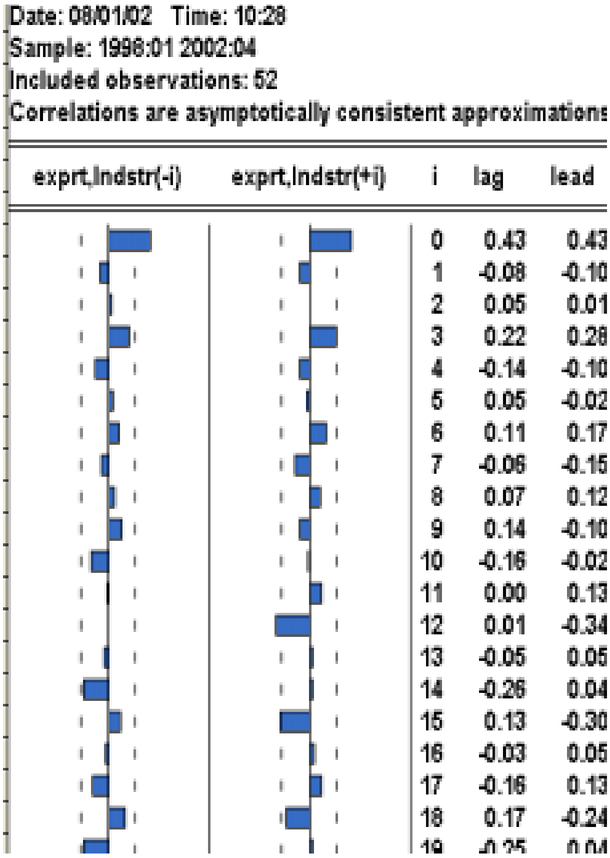
Several studies in the last few years (Diebald and Rudebusch, 1996; Chauvet, 1998; and especially Kim and Nelson, 1999) tried to overcome these disanvantages, combining the dynamic model of unobservable common factor, as a composite index, on the one hand, with the idea of switching between the two regimes, expansion and recession, on the other.

Smith and Summers (2001, 2002) and Batchelor (2001) contain detailed discussions of the implementation of the integrated approach in several advanced economies. The findings indicate that while in the US, Australia and France the new approach added to the explanatory power of the business cycle and improved the ability to predict them in the short term, in the UK, Japan and Germany the integrated model made very little additional contribution to the explanation of economic activity and its turning points.

In Israel the central bank, in a paper by Melnick and Golan (1992), adopted Stock and Watson's methodology for compiling the state-of-the-economy index. Eventually four series were selected as index components: the index of manufacturing production (excluding diamonds); the index of large-scale retail trade; goods imports, excluding capital goods, fuel and diamonds; and advertised job vacancies (later changed to the number of employee posts in the business sector). The composite state-of-the-economy index was constructed as the weighted sum of current and lagged growth rates of its components. Following the Stock and Watson's methodology, Melnick (1992) also tried to fit a structural model that would provide a short-term forecast of the composite index via leading indicators, but the results of the monthly forecasts were not accurate enough.

At the end of 1990s it became clear that the composite index was encountering difficulties in identifying business cycles and in locating the turning points in real time. Several improvements were introduced, therefore. A series of trade and services revenue replaced the series of large-scale retail trade, thus correcting the bias arising from the trend towards the multiple retail outlets (chain stores), and the inclusion of a services index, representing the important services industry, also served to improve the overall index. The weights assigned to the various components were recalculated, according to the updated sample. However, several flaws remained. The index rose at times which were generally viewed as being periods of recession, such as 1996–98 and the last one, starting with the outbreak of the *intifada* in October 2000: during two first years of this recession the quart of measured index values were positive. Finally, it continued to exhibit high volatility, making it difficult to spot turning points on the business cycle; and the lack of consistency remained a problem, as did the many and frequent revisions carried out on its components.

Table 1. The cross correlations between the index of exports goods and the industrial production index



```
Date: 08/01/02   Time: 10:28
Sample: 1998:01 2002:04
Included observations: 52
Correlations are asymptotically consistent approximations

  exprt,Indstr(-i)   exprt,Indstr(+i)    i    lag    lead

                                         0   0.43   0.43
                                         1  -0.08  -0.10
                                         2   0.05   0.01
                                         3   0.22   0.28
                                         4  -0.14  -0.10
                                         5   0.05  -0.02
                                         6   0.11   0.17
                                         7  -0.06  -0.15
                                         8   0.07   0.12
                                         9   0.14  -0.10
                                        10  -0.16  -0.02
                                        11   0.00   0.13
                                        12   0.01  -0.34
                                        13  -0.05   0.05
                                        14  -0.26   0.04
                                        15   0.13  -0.30
                                        16  -0.03   0.05
                                        17  -0.16   0.13
                                        18   0.17  -0.24
                                        19  -0.25   0.04
```

The above difficulties led the authors to think about a model with non-linear properties. The new model follows one of Kim and Nelson (1999). It preserves the State-Space framework, but adds a non-linear term - the expected deviation from a long-run growth - that depends itself on the state of the economy (i.e., expansion or recession). The state cannot be measured and is presumed to follow a two-regime Markov chain. When there are two possible regimes, expansion and recession, there are four transition probabilities to be estimated: the probability to switch from expansion to recession, and from recession to expansion, to remain in recession, and to remain in expansion. The probability of recession is the simple sum of the probability of remaining in recession and the probability of moving from expansion into recession. The extra information on the probability of a recession over and above the data of the composite index enables the turning point in the business cycle to be identified more sharply for two reasons: 1) the calculation of the probability of a recession makes use of more information (throughout the sample period) than does the calculation of the index itself, so that the current business cycle can be compared, for example, to other cycles. 2) The monotonic nature of the normal distribution used to calculate the probability smoothes fluctuations and shocks.

In addition to this change in the new index, various improvements have been introduced regarding its individual components. As Israel's economy is a small, open one, a fifth series, that of goods exports (excluding agriculture, fuel, diamonds, and ships and aircraft), has been incorporated into the composite index. This series is simultaneously correlated with the other four series, and its inclusion raises the maximum likelihood criterion (table 1).

The business-sector employment rate, having very low variance, was expanded to the order of the industrial production variance,[4] by $\sqrt{10}$ correction factor. We also explore

---

[4] The practice of correcting the data of series incorporated in the composite index is accepted in other countries, because the imbalance of the variance of different components of the index is likely technically to grant undue weight to a component with a lower variance at the expense of other components with relatively high variances.

the fact, pointed out by Stock and Watson (1989) and by Kim and Nelson (1998), that the labor market indicator may not be exactly coincident, but “slightly lagged”, therefore three employment lags were included in the model. Furthermore, to avoid outliers among the import and export fluctuations, a ceiling of two standard deviations (within a moving 5-years window) was imposed.

3. The model

The composite index represents the non-observable underlying factor that causes the mutual fluctuation of indicators that depict the state of the economy. This common factor develops according to a stochastic autoregressive process.

The selected indicators that characterize the state of the economy are called the components of the index, and they are measured by their rates of change (or log differences). In the version herein, the index consists of five components: the index of manufacturing production, *i*; the imports index, *m*; the sales revenue index, *r*; employee posts in the business sector, *e*; and the index of exports goods, excluding agriculture, *ex*. The current rate of change in the composite index, ΔC_t, is derived from a system of six simultaneous equations that describe the dynamics of the five components of the index and also the development of the index itself:

$$\Delta C_t = \varphi_1 \Delta C_{t-1} + \varphi_2 \Delta C_{t-2} + \varphi_3 \Delta C_{t-3} + \mu_{S_t} + \upsilon_t \tag{1}$$

$$i_t = \gamma_0^{(i)} \Delta C_t + \psi_1^{(i)} \varepsilon_{t-1}^{(i)} + \varepsilon_t^{(i)} \tag{2}$$

$$m_t = \gamma_0^{(m)} \Delta C_t + \psi_1^{(m)} \varepsilon_{t-1}^{(m)} + \varepsilon_t^{(m)} \tag{3}$$

$$r_t = \gamma_0^{(r)} \Delta C_t + \psi_1^{(r)} \varepsilon_{t-1}^{(r)} + \varepsilon_t^{(r)} \tag{4}$$

$$e_t = \gamma_0^{(e)} \Delta C_t + \gamma_1^{(e)} \Delta C_{t-1} + \gamma_2^{(e)} \Delta C_{t-2} + \gamma_3^{(e)} \Delta C_{t-3} + \psi_1^{(e)} \varepsilon_{t-1}^{(e)} + \psi_2^{(e)} \varepsilon_{t-2}^{(e)} + \varepsilon_t^{(e)} \tag{5}$$

$$ex_t = \gamma_0^{(ex)} \Delta C_t + \gamma_1^{(ex)} \Delta C_{t-1} + \gamma_2^{(ex)} \Delta C_{t-2} + \gamma_3^{(ex)} \Delta C_{t-3} + \psi_1^{(ex)} \varepsilon_{t-1}^{(ex)} + \psi_2^{(ex)} \varepsilon_{t-2}^{(ex)} + \varepsilon_t^{(ex)} \tag{6}$$

Each of the index components is explained by the common factor ΔC_t, throughout the elasticities $\gamma_0^{(i)}$, $\gamma_0^{(m)}$, $\gamma_0^{(r)}$, $\gamma_0^{(ex)}$, $\gamma_0^{(e)}$, but also contains an idiosyncratic factor, described by a moving-average process of the first or second order. The employment equation (5) , augmented by three lags of ΔC_t, captures the slightly lagged reaction of the labor market to the cycle. The lags are also added to the export equation (6), in order to catch the various delay of export response.

It is assumed that the residuals $\varepsilon_t^{(i)}$, $\varepsilon_t^{(m)}$, $\varepsilon_t^{(r)}$, $\varepsilon_t^{(ex)}$, $\varepsilon_t^{(e)}$ are independent measurement errors.

Therefore, the system (1)-(6) also includes the non-observable variable, the change in the composite index, ΔC_t ; its parameters are not known either.

The analytical framework for such a system, widely applied by Harvey (1989) and others in econometrics, is known as State Space Representation and consists of two matrix equations:

$$Y_t = HZ_t + \varepsilon_t \qquad (7)$$

$$Z_t = \mu + FZ_{t-1} + \eta_t \qquad (8)$$

The measurement equation (7) links the vector $Y_t = (i_t, m_t, r_t, e_t, ex_t)$ of index components and the state vector Z_t, the first element of which is the index change $\varDelta C_t$. The transition equation (8) shows how the state vector evolves from time *t-1* to time *t*. For details refer to appendix B.

Notice, that the state space form has no unique solution that mostly depends on the inferences about the system parameters.

For the linear model, like dynamic coincident model of Stock and Watson (1989), an iterative method was developed, combining at each iteration an estimation of the state vector (by Kalman filter pass), conditional on given parameters, and parameters estimation, by maximization of the likelihood function (EM algorithm).

We use here a non-linear model, motivated by Kim and Nelson's (1995), which involves a non-linear intercept μ_{S_t}, defined as follows:

$$\mu_{S_t} = \mu_0(1 - S_t) + \mu_1 S_t \ ,$$

where the binary S_t variable characterizes the state of economy (regime): $S_t = 1$ in expansion, and $S_t = 0$ in a recession.

Thus, μ_{S_t} takes the value of μ_0 or μ_1 according to the regime.

In other words, the system provides two average rates of growth (one in the recession and another in expansion), which deviate around the long-term growth rate that remains indeterminate. Another issue is that the average rate in the recession and the average rate in expansion do not necessarily sum up to the long-term average rate, as the two regimes are not symmetrical.

The regimes S_t themselves are unobservable, but the probability of switching from one to the other can be estimated. Also assumed, that the probability that S_t equals some particular value (0 or 1) depends on the past only through the most recent value S_{t-1}.

Thus, for this two-regime Markov chain process, four transition probabilities can be estimated: $\Pr(S_t = 0 | S_{t-1} = 0)$ - the probability of remaining in a recession if the economy is already in a recession; $\Pr(S_t = 0 | S_{t-1} = 1)$ - the probability that the recession is followed by expansion; $\Pr(S_t = 1 | S_{t-1} = 0)$ - the probability of switching from expansion to recession, and $\Pr(S_t = 1 | S_{t-1} = 1)$ - the probability of remaining in expansion. Obviously, the probability of being in a recession at time *t* relates to two possibilities: that of remaining in a recession when the economy was in one, and that of switching from expansion to recession, i.e. the corresponding probabilities are summing up:

$$\Pr(S_t = 0) = \Pr(S_t = 0 | S_{t-1} = 0) + \Pr(S_t = 0 | S_{t-1} = 1)$$

The probabilities depend on the model residuals $\varepsilon_t^{(i)}$, $\varepsilon_t^{(m)}$, $\varepsilon_t^{(r)}$, $\varepsilon_t^{(ex)}$, $\varepsilon_t^{(e)}$, and also on the past probabilities.

The estimation idea implicitly exploits the properties of recursive filter, which, unlikely the regression technique, allows inconsistency between the intercept and the estimated average.

Actually, since intercept μ_0 or μ_1 is assumed, the average state $\overline{Z} = (I - F)^{-1} \mu$ is immediately set, where *I* is the unit matrix. Upon these inferences, four possible scenarios can be handled: the filter starts with the μ_0 intercept and recovers the state vector with corresponding mean $\overline{Z}(0)$; the filter starts with the μ_1 intercept and leads to the corresponding average state $\overline{Z}(1)$. With it, the combination of μ_0 with the average state $\overline{Z}(1)$, and vice versa, are also legitimate. Applying the Kalman filter recursion, we get the state vectors, consistent with the inferences made. Recalling that the composite index change ΔC_t constitutes the first element of the state vector, we understand that there are four different $\Delta C_t \mid (S_t = i, S_{t-1} = j)$ indices, relating to the same observation, but obtained under different assumptions.

As soon as the four state vectors become observable, this enables us to calculate the residuals. By now, we can deal with: $\varepsilon_t(S_t = 1, S_{t-1} = 0)$ - deviations in the case of transition from recession to expansion; $\varepsilon_t(S_t = 0, S_{t-1} = 1)$ - deviations in the case of transition from expansion to recession; $\varepsilon_t(S_t = 1, S_{t-1} = 1)$ deviations when remaining in expansion; and $\varepsilon_t(S_t = 0, S_{t-1} = 0)$ when remaining in a recession.

The intuition behind it is that without loss of generality, we assume the regime switch at each observation, produce the suitable state vector and then check by the system deviation whether the switch has actually happened.

For every vector of residuals η_t the normal density function can be calculated by:

$$f_t(Y_t | S_{t-1} = i, S_{t-1} = j, \Theta_{t-1}) = 2\pi^{-\frac{N}{2}} \det(F)^{-\frac{1}{2}} \exp\{-\frac{1}{2} \eta_t'(S_t = i, S_{t-1} = j) F \eta_t(S_t = i, S_{t-1} = j,$$
$$i, j = 0,1$$

that is conditional on the index components (vector Y_t), on the regimes S_t and S_{t-1} between which the transition takes place, on the coefficients F and on the estimation results up to time t-1 (matrix Θ_{t-1}).

The probability of a switch from the regime $S_{t-1} = j$ to the regime $S_{t-1} = i \quad (i, j = 0,1)$ given all the information Θ_t up to time t is

$$\Pr\{S_t = i \mid S_{t-1} = j, \Theta_t\} = \frac{f_t(Y_t \mid S_t = i, S_{t-1} = j, \Theta_{t-1}) \Pr\{S_t = i \mid S_{t-1} = j, \Theta_{t-1}\}}{\sum_{i,j} f_t(Y_t \mid S_t = i, S_{t-1} = j, \Theta_{t-1}) \Pr\{S_t = i \mid S_{t-1} = j, \Theta_{t-1}\}}$$

In other words the probability is calculated recursively based on probabilities estimated up to the previous observation and normalized over the whole sample up to time t.

To start the recursion, intuitively chosen steady-state (time independent) transition probabilities are employed at the first observation, i.e.,

$$\Pr(S_t = i, S_{t-1} = j \mid i, j = 0,1) = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} .$$

Finally, we aggregate partial indices $\Delta C_t \mid (S_t = i, S_{t-1} = j)$, conditional on assumed regime transitions, into a resulting one, by weighting them by their respective probabilities:

$$\Delta C_t = \sum_{i,j} (\Delta C_t \mid S_t = i, S_{t-1} = j) \Pr\{S_t = i \mid S_{t-1} = j, \Theta_t\} .$$

As every estimate is dependent on the coefficient matrices in F, H, and μ, we iterate between the estimation step, which produces the state vectors and vectors of corresponding probabilities, conditional on the parameters, and the maximization step, which improves the parameters according to the likelihood function L

$$L = \ln \sum_{t=1}^{T} \left(\sum_{i,j} f_t(Y_t \mid S_t = i, S_{t-1} = j, \Theta_t \right)$$

Notice that at the maximization step we adopt the state vector Z_t as a regular series, because it became observable after the Kalman filter pass. This enables to solve the system (1)-(6) with respect to the parameters, via the SUR technique.

The following summarizes the calculations performed at each iteration:

- four passes of the Kalman filter, to calculate four indices in accordance with four regime-switching assumptions;
- derivation of four series of transition probabilities, in accordance with the four vectors of system deviations;
- weighting of the four partial indices by appropriate probabilities to obtain one composite index giving the state of the economy at time *t*;
- estimating of the new parameters by maximizing the likelihood function *L*;
- the next iteration starts with the new parameters, until the process converges, i.e., the parameters stabilize or there is no further improvement to the likelihood function *L*.

We omit here other technical details of estimation, such as probabilities smoothing, despite their importance. These may be found in Kim and Nelson (1998).

To conclude the discussion of the model, a distinction is drawn between the method of estimating the parameters and the month-by-month calculation of the composite index. The old index, based on the linear model, was currently calculated with fixed weights, derived on a one-off basis, and updated when necessary.

The basket of weights $W_{t-p,k}$ for calculating the composite index in the form

$$\Delta C_t = \sum_{p,k} W_{t-p,k} Y_{t-p,k} (k = 1,..4) \qquad (9)$$

is equal in the linear model to:

$$W = (I - KH)F \qquad (10)$$

and depends on the *F* and *H* matrices of parameters,

where

$$K = [\operatorname{cov}(Z)H`(H \operatorname{cov}(Z)H` + \operatorname{cov}(\eta)^{-1}]$$

is the Kalman gain.

Also in the new index, we refer to the system parameters as one-off estimates. They seem stable enough to be used for several years. However, the basket of fixed components weights is already insufficient for current monthly calculation, because it also requires update of the “basket of probabilities.”

4. The estimation results

The main sample covered the period from September 1992 to October 2002. The components of the index were calculated from the following monthly indicators (all, excluding the employment series, at fixed prices) published by the Central Bureau of Statistics (CBS): the index of manufacturing production; the index of imports (without capital goods); the index of revenue of trade and services (starting only at 1991); the index of business-sector employee posts; and the index of exports goods (excluding agriculture).

In order to get the longer cycle estimates, we used the retail trade index[5] instead of the sales revenue index for the wider sample, covering the period from 1975 to date. Thus a long-term series of the composite index and the probability of recession have been obtained and further overlaid by more accurate estimates based on the sales revenue index since September 1992.

The initial parameters values were obtained from the linear Stock and Watson model for the same sample, i.e., the estimation started from the point to which the linear model converged.[6] The starting values of the transition probabilities were set at

$$\begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} = \begin{bmatrix} 0.9 & (1-0.9) \\ (1-0.8) & 0.8 \end{bmatrix}$$

and remained almost the same.

Appendix C shows the estimated parameters. The average decline of the growth rate in the recession periods was obtained as $\mu_0 = -0.5\%$, when the average rise of growth during the expansion is about $\mu_0 = 0.6\%$.

Table 3. The weights of the new index components

Lag	Industry	Imports	Revenue	Employment	Exports
0	0.26	0.07	0.23	0.25	0.09
1	-0.10	0.04	0.07	-0.10	0.05
2	0.08	0.00	-0.05	-0.08	0.00
3	0.04	0.01	0.03	0.12	-0.04
4	-0.01	0.00	0.00	0.04	0.00
Total	0.26	0.12	0.28	0.23	0.10

---

[5] This was one of the components of the Bank of Israel's index used until 2001, when it was replaced by the sales revenue index.

[6] We appreciate Kim and Nelson's (1999) recommendation to start the estimation with the parameters, which found optimal for the linear model, rather than from a completely "free" guess. This system appears more sensitive to the initial values of the parameters than is the linear model: when the initial values are too far from the optimum area, the system reacts by flattening the probabilities

Table 4. The weights of the old index components

Lag	Industry	Imports	Revenue	Employment
0	0.31	0.10	0.26	0.46
1	-0.02	0.02	-0.04	-0.11
2	0.01	0.00	0.01	0.03
3	0.00	0.00	0.00	-0.01
Total	0.26	0.12	0.22	0.37

As explained in the description of the model, the new index does not have a constant basket of components, as the basket of probabilities changes from one observation to the next. It is nevertheless of interest to compare the composition of the new index to that of the old one. The weights of the new index components relevant to the latest period were therefore derived purely for illustrative purposes in exactly the same way as they are calculated in the linear model.[7]

Tables 3 and 4 enable the comparison. Bearing in mind that normalizing the weights to a total of 100 percent in the new index is carried out relative to the employment component corrected by $\sqrt{10}$, it can be seen that the contribution of the manufacturing, imports and turnover components is slightly reduced to make room for the new component, exports, and the contribution of the employment component increased relative to its weight in the old index.

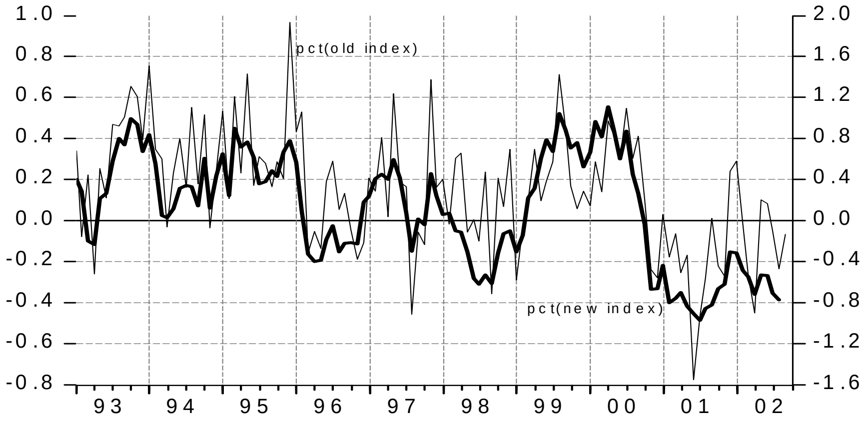
Another interesting aspect of the new index is the division of the signs of the weights according to the export lags: the first three export lags are included with positive weights, serving to smooth the composite index because of the fluctuations in exports from month to month.

A first examination of the findings related to the index immediately indicates two advantages of the new index over the old one. The monthly fluctuations in the new index are much smaller than those in the old one (Figure 1). The decline in the composite index since the latter part of 2000 and that in the period prior to the beginning of 1999 are indicated far more clearly by the new index. Another result yielded by the findings is that the average long-term rise is significantly lower in the new than in the old index, 0.05 percent compared with 0.35 percent respectively.

---

[7] We refer to equations (9)-(10) in Section 3.

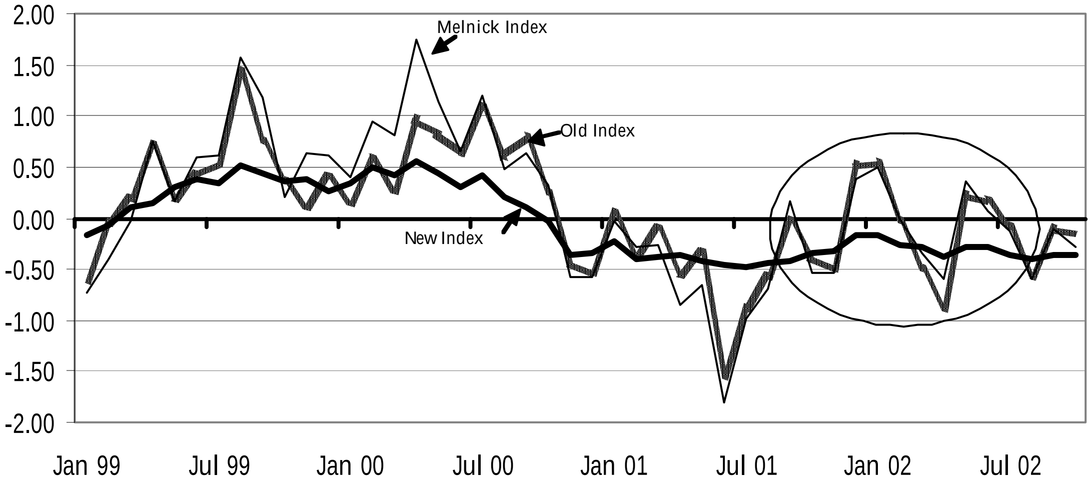
Figure 1. Monthly changes of the old (the right scale) and new composite indices



The effect of smoothing fluctuations in the index as a result of changing the method of estimation is also apparent if the comparison is made using the Melnick version of the composite index (Melnick 2002), which is based on the same components as is the Bank of Israel's old index and is also calculated from the Stock and Watson linear model but with slightly different weights.

Figure 2 illustrates the volatility problem with respect to two indices - calculated by the linear model - whose components follow wide fluctuations due to the lack of a control mechanism. This was referred to in the previous section, where it was explained that only the new version provides such a mechanism, with the help of reweighing by the transition probabilities. Thus in December 2001 and January 2002 those two indices registered rises of about 0.5 percent, and in May and June they rose by 0.2 percent, when the economy was already showing clear signs of recession.

Figure 2. The volatility of the linear indices



These observations have undergone several revisions, and remain exceptional related to the nature of the period. The new index, on the other hand, qualifies the last recession by the most consistent way.

The new estimates thus provide the probability of recession as part of the model.
Figures 3 and 4 depict the results, when the probability of a recession illustrates a monotonous mirror image of the index dynamics.

In order to obtain a formalized classification of the business cycle we applied a version of the Stock and Watson (1993) criterion of a recession. Two recessionary patterns have been defined: in the first, the probability of recession is greater than 0.5 for six consecutive months. Another kind of pattern is possible, when the probability exceeds 0.5 at least for seven out of eight months, including the first and last. The first column of Table 5 and the shaded areas in the figures represents the results of this classification. A similar criterion was applied to the index change too[8] (the second column of Table 5).

Figure 3. The change of the new index

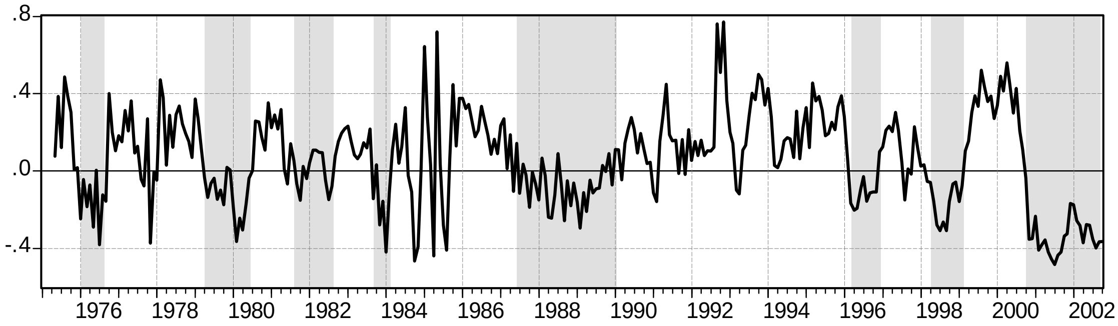
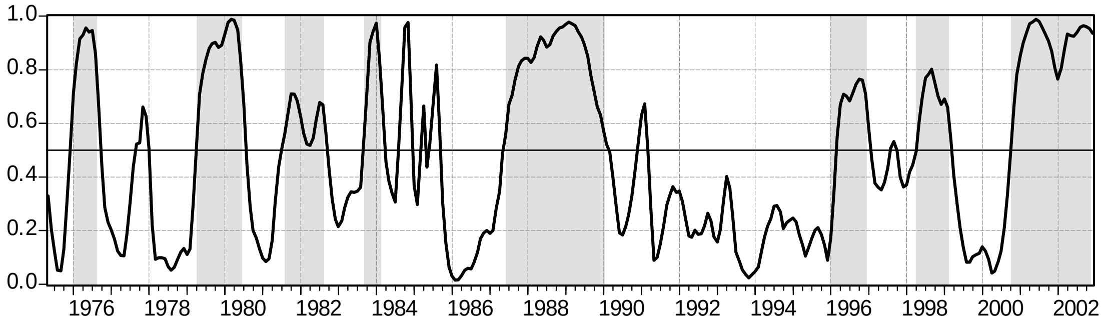


Figure 4. The probability of Recession



---

[8] A recession period classified by the index change defined, when it falls below zero for 6 out of 9 consecutive months. However, the probability, as it is trend free, seems a more suitable measure for the classification, because it does not require gradual change of the threshold during the sample.

The probability of recession seems to mark all broadly acknowledged recessionary periods in Israel in the sample period: in 1979–80, when the government adopted a tight policy; the recession in 1987-1989 due to tight monetary policy and the emerge of the first *intifada*; and finally the recessions in 1996, 1998–99 and the current one that started towards the end of 2000.

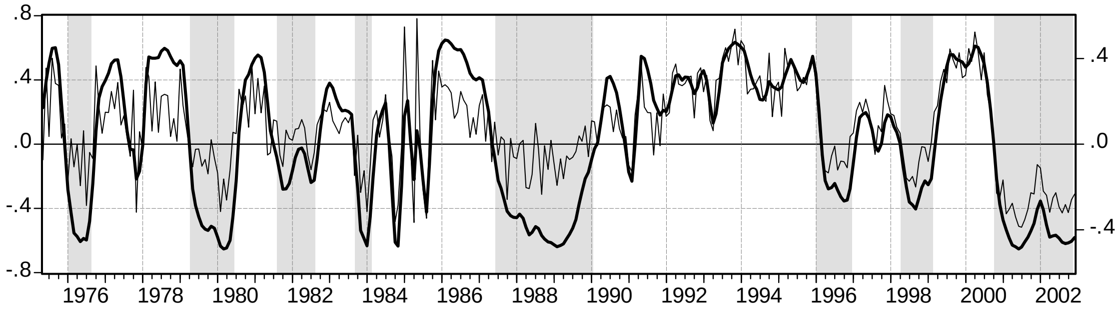
The picture of the business cycle in general is clearer when it is derived from the probability of recession rather than from the new composite index. Note that the selected recession periods classified by the probability of a recession is not sensitive to slight changes in the criterion of recession, while this is not true when using the index change. According to the last criterion we get only 5 periods of recession compared to 8 recession periods obtained by the probability criterion.

Table 5. Classification of recessionary periods for 1975- 2003

By probability of a recession		**By new composite index change**	
Period	**Duration** (months)	**Period**	**Duration** (months)
Jan 1976- Sep 1976	9		
Apr 1979- Jul 1980	16	Apr 1979- May 1980	14
Aug 1981- Sep 1982	14		
Sep 1983- Mar 1984	7		
Jun 1987- Feb 1990	33	Apr 1988- Aug 1989	13
Mar 1996- Jan 1997	11	Mar 1996- Nov 1996	9
Apr 1998- Mar 1999	12	May 1998- Jan 1999	9
Oct 2000- …	…	Nov 2000- …	…

Figure 6 depicts the standardized probability of expansion (0.5 *minus* the probability of a recession) alongside the index change. The shaded areas show all the recession periods, according to the probability, part of them not identified by the index change (table 5).

Figure 6. The New Index and the Standardized Probability of Expansion (bold line, the right scale)

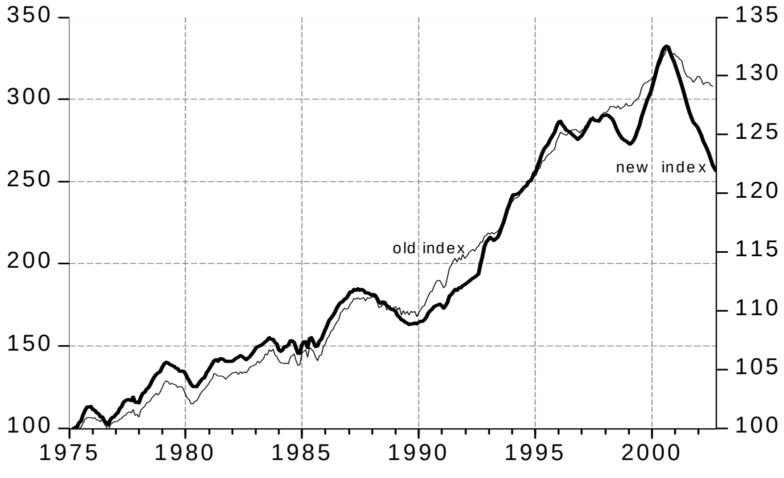


Now let us consider the index level. Bearing in mind that the level of the index is not defined by the model, we can only derive it from the index change, by :

$$C_t = C_{t-1} + \Delta C_t \quad . \tag{11}$$

As soon as the new index is measured in deviations from the long-run growth rate, it does not incorporate the major part of the component trends. Therefore, the sharp difference in scale of the new and old indices' levels (figure 7) is not surprising. Thus, from 1975 to its peak level the new index rose by only 35 percent, while the old index soared by 230 percent. Apparently the new scale of the level of the index outperforms the cyclical pattern of economic activity, while the old index level tends to reflect the GDP upward trend.

Figure 7. The level of the new (the right scale) and the old indices



Having a very small positive slope, the new index level becomes inconsistent with economic policy indicators, sensitive to the scale. For example, the regression of money demand dependant on the new index level loses the expected elasticity. Aware of this inconsistency, we distinguish between the diagnostic purposes, when the original (11) index scale is useful, on the one hand, and different frameworks, on the other, which can require a scale compatible with the GDP growth.

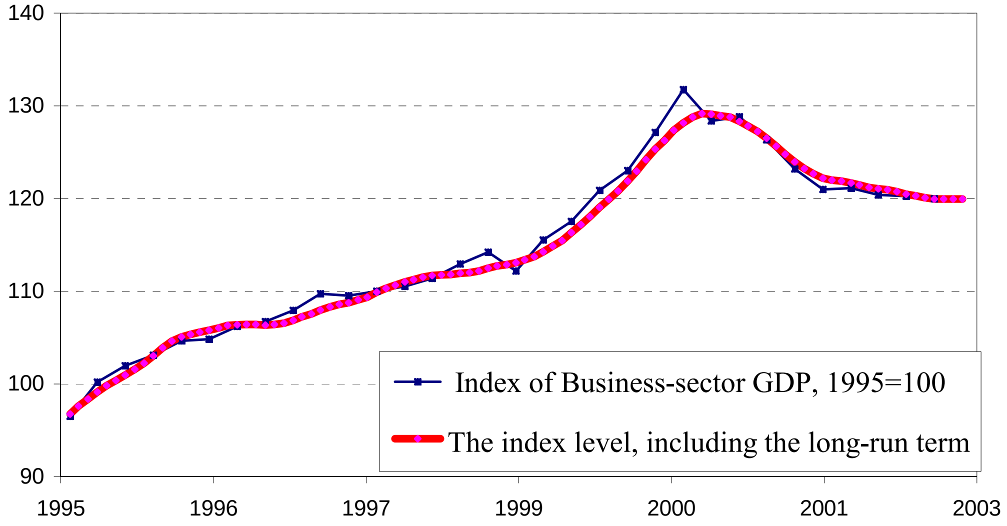
Answering users' requests, looking for an index which would include also a long-run growth term, we built up an "expanded-scale" index, computed in such a way,

$$C_t = C_{t-1} + \Delta C_t + \delta_t \quad , \qquad (12)$$

where a long-run growth term δ_t , added to each observation, can be obtained as a Henderson curve of the linear model with weights given in Table 3.

Figure 8 demonstrates the "restored" index level, when the trend component, deleted by construction, is reintroduced. As can be seen, it closely reflects the dynamics of business GDP product, advancing it more than by quarter.

Figure 8. The "restored" index level, including long-run growth term, alongside the quarterly index of business sector GDP.



However, it should be taken into account that the Henderson curve estimates cannot be precisely computed at the end of the sample because of one-side window smoothing. For this reason, the index change, based on (12), is less reliable for the current diagnostics than the proposed new index, and should rather be implemented in historical analysis and regressions.

The new index can also be compared to some external criterion, such as National Accounts indicators that are accepted as representing economic activity and available quarterly after several months. Table 6 presents the coefficients of correlation between these indicators, compared as the quarterly averages, as well as during the quarter (end of quarter compared to the end of the previous quarter[9]). It can be shown that the new index is more highly correlated with the National Accounts indicators, than is the old index. Its advantage becomes more clear, when we compare within the "during the period" domain, which requires indices, computed at monthly frequency.

Table 6. Coefficients of correlation between three versions of the composite Index and Indicators from the National Accounts

National Accounts indicators	Quarterly averages			During the quarter		
	Old index	Melnick Index	New index	Old index	Melnick Index	New index
Business-sector gross product	0.78	0.81	0.78	0.44	0.57	0.86
Private consumption	0.71	0.69	0.64	0.53	0.64	0.84
Exports	0.69	0.61	0.70	0.55	0.65	0.75
Imports (excl. defense imports)	0.56	0.54	0.57	0.60	0.59	0.66
Final uses (excl. defense imports)	0.82	0.81	0.79	0.57	0.66	0.85
Business-sector employees	0.12	0.28	0.29	0.11	0.21	0.52
Unemployment rate	-0.23	-0.31	-0.12	-0.00	-0.08	-0.25

5. Real-time estimates and the problem of revisions

After it is published the composite index is revised for several reasons. One is the fact that the CBS updates the data on which the index is based. The CBS estimates of certain observations stabilize, if at all, only after several additional figures have been published.

For example, the rate of change of the index of manufacturing production, seasonally adjusted, reported by the CBS for April 1998 has been published more than fifty times since then, and only stabilized around the average estimate for that period (-1.5 percent) in the last twelve publications, with the standard deviation of that estimate at about 0.6 percent. As this figure of manufacturing is a focal component of the composite index, its updates lead to revisions in the index itself. This problem is particularly acute in the first publication of the manufacturing index as it is based on only half of the sample of manufacturing companies.

The addition of a new observation to the end of a series every month, that entails a re-estimation of the seasonally adjusted component throughout the series, is another cause of revisions to the components of the index. The composite index thus needs to be updated continually, incorporating changes even to observations relating to months long before the current month.

---

[9] The National Accounts estimates at the end of quarter were made by interpolation of neighbors quarters data.

The absence of certain data when the index is being calculated is another reason for its revision. Of the five components of the index, only two, the import and export indices, are known coincidentally, while for manufacturing production and the sales revenue indices the changes from the last month's index have to be guessed, and for the employment index, the change over two months.

It can be seen from the above that the problem of revisions of the index is an objective one, and characteristic of Israel's economy; it is reasonable to expect that a synthetic indicator of the state of the economy, such as the composite index, should be smoother and more stable than its components.

The extent of the revisions to four important indicators were compared: (1) the index of manufacturing production, as an essential component of the composite index on the one hand, and the most appropriate to be updated on the other; (2) A composite index constructed by weighting its components as in Stock and Watson, such as the Bank of Israel's old index; (3) A composite index of regime switching such as that proposed; and (4) The probability of a recession, calculated together with the new composite index.

The starting point for the comparison is that every observation for month *t* in any of the indicators undergoes revision for the following publication, so that several "generations" of publications accumulate. These estimates are dispersed around the average to which the latest publications generally converge. This posterior distribution is generally characterized by its standard deviation.

Table 7. The Standard Deviation of Indices Relating to the Same Month

Period	Manufacturing production index σ_i	Old composite index $\sigma_{\Delta S}$	New index $\sigma_{\Delta C}$	Probability of recession $\sigma_{\Pr ob}$
1998	0.94	0.25	0.10	0.08
1999–2000	0.66	0.20	0.18	0.11
2001–2002	0.30	0.10	0.04	0.02
Average	0.58	0.18	0.13	0.08

Table 7 shows the revisions that these indicators underwent in the follow-up period, i.e., from 1998 to the present. For each indicator the standard deviation was measured for each collection of publications that give an estimate for the same month. The months were sampled randomly. A general picture (the expected revision) emerges from the standard deviation thus calculated, and also for different periods, if it is assumed that the size of the updates differs between expansion and recession periods.[10] Another reason for dividing the period into sub-periods is the gradual improvement in the quality of the

---

[10] This is a reasonable assumption if the heteroscedasticity effect exists, i.e., the size of the revision is correlated with the data, so that in expansion the size of the revisions also rises.

estimate by the CBS of the index of manufacturing production, which is reflected in the table as revisions, which decrease over time.

Therefore, from the revisions aspect the probability of recession provides the most stable estimates. Evidently, the expected revisions in the above four indicators can be arranged in the following order:

$$E(\sigma_{\mathrm{Pr}ob}) \leq E(\sigma_{\Delta C}) < E(\sigma_{\Delta S}) < E(\sigma_i) .$$

Further analysis shows that this order holds not only on average, but also in most cases: in 88 percent regarding probability, in 75 percent regarding the new index, and in every case regarding the old index.

The question arises, what is the mechanism that ensures that this order holds? The answer lies in the method of calculating the new index. In practice, the probability of any particular regime, calculated from the residuals in the model, constitutes a control mechanism for the fluctuations of the components and assigns weights to the fluctuations corresponding to their randomness. Weighting the four possible indices by these transition probabilities provides an additional filter that does not exist in the estimate of the old index.

Stated differently, the expected revision, that declines from the manufacturing production index to the old composite index, and from that to the new index and thence to the probability of recession is explained by the number of filters used in the estimation process. Thus, in calculating the old index, its components are weighted and the result is smoothed by a three-month moving average. This ensures that the expected revision of the result does not exceed that of the components. The new index not only weights the components but also adjusts noise in their fluctuations via the appropriate probabilities, thereby ensuring a more stable estimate.

Another point that arises in this context is the ability of any indicator to identify a turning point in the business cycle in real time, or what picture of the business cycle is painted by any index at its first estimation, compared with the picture seen after a longer period? To answer this question, three relatively recent episodes in Israel’s economy were examined: the recession in 1998–99, the expansion that started in the second half of 1999 and the current recession that started towards the end of 2000.

Figure 9. The probability of recession: the real-time estimate and the tenth revision

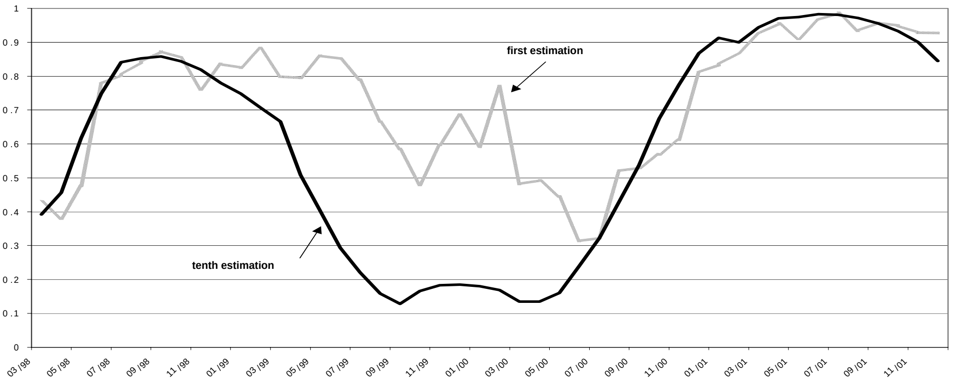


Figure 10. The new index: real-time estimate and the tenth revision

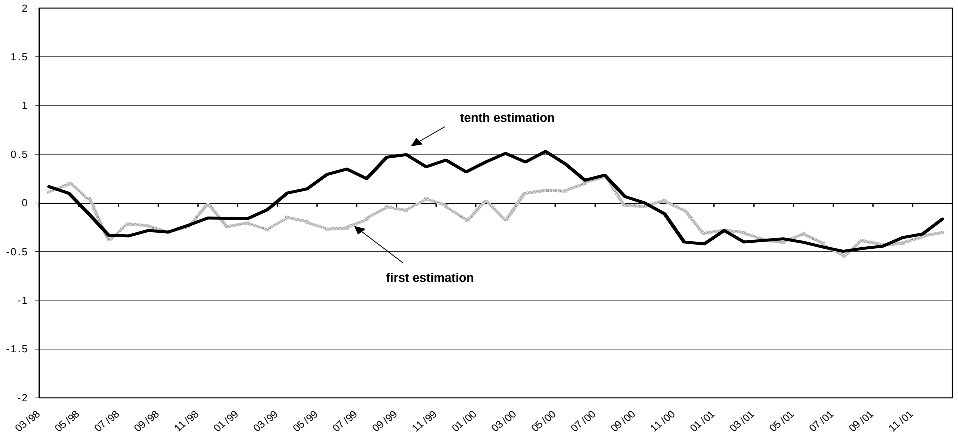
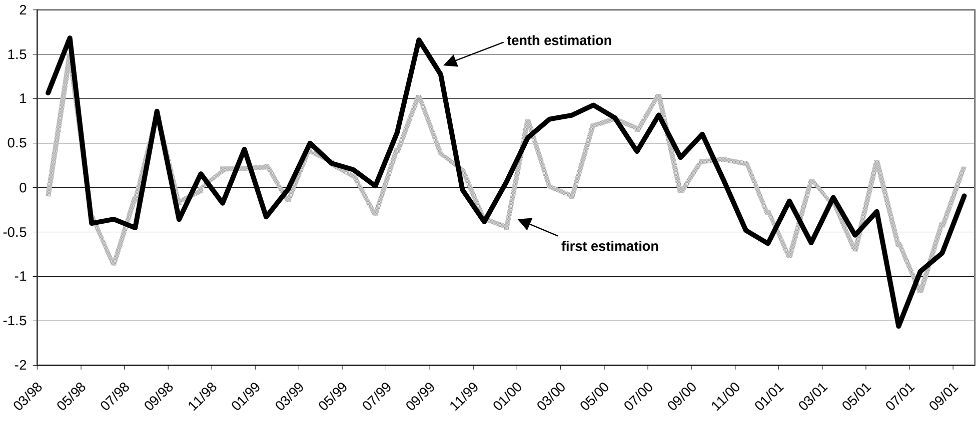


Figure 11. The old index: the real-time estimate and the tenth revision



Figures 9 to 11 show the situation as described by the three indicators - the old composite index, the new index, and probability of a recession - as would appear in the first publication of the indicator related to a particular month, and as would appear in the next ten monthly publications relating to that same month. The figures lead to the conclusion that the recession probability identified the start of the two recessions more quickly and more definitely than did the index itself. Although the most recent recession that started in October 2000 was identified with certainty by the new index and by the probability factor about two months after it had started, the latter gave the clearer picture. With regard to the previous recession too, in 1998-1999, the real-time probability of a recession (when first published) showed that the economy would be in a recession throughout that period, while the new index indicated greater volatility.

Turning to the identification of the start of growth in July 1999, the probability indicated in real time a change in activity, unlike the new index. In the next few months, however, there were indications - derived from the publication of negative indices relating to components of the composite index at that time - of a return to the recession; these components were later revised. Only after ten publications did the new index, and specifically the probability factor, describe high-tech growth. The old state-of-the-economy index, on the other hand, encountered difficulty in identifying these occurrences not only in real time but up to the present, resulting from secondary fluctuations of the index, and the retrospective picture becomes clearer only by analyzing the level of the index and not the rate of change.

6. Conclusions

This paper examines a new method of calculating a composite state-of-the-economy index in Israel that incorporates the Markov chain into the State Space framework. This approach has gained acceptance in the last few years as describing the structure of the business cycle: its main advantage is that unlike previous ones there is no need to assume symmetry between periods of contraction and periods of expansion of economic activity. The model herein simultaneously estimates the composite index and the probability of being in a recession.

In addition to this new approach to the calculation of the composite index, the components of the index have been improved: the goods exports series has been included in the index as appropriate to Israel's small open economy, and the lack of simultaneity (i.e., the delayed reaction) of the employment component has been taken into account. The new index does not suffer from the drawback of the old index in that it distinguishes business cycles in general, and not only classical ones.

The results of the estimation together with a broad range of tests presented in the paper show that the new composite index and in particular the probability of a recession enable a clearer picture of the business cycle to be obtained than that painted by the old index. Moreover, the probability of a recession gives a more accurate description of the business cycle than does the new index itself, since it captures the turning points in the business activity faster. The recession probability succeeds in identifying the start of a recession immediately, and even more clearly two months later. The growth period in high-tech was also identified more clearly by the probability factor (retrospectively) than by the new index itself. Another advantage of the new index, and especially of the probability factor, over the old index is that it is less volatile and subject to fewer revisions.

References

Burns, A and Mitchell , C. (1946), “Measuring Business Cycles” , *NBER Studies in Business Cycles*, No 2, NBER ,New York.

Batchelor, R. (2001), “Confidence Indexes and the Probability of Recession: A Markov Switching Model”, *India Economic Review - Special Issue on Analysis of Business Cycles*, 107-124.

Boehm, E and Moore G.H. (1984), “New Economic Indicators for Australia, 1949-84“, *Australian Economic Review*, 4-th Quarter, 34-56.

Boehm, E (2001) “The Contribution of Economic Indicator Analysis to Understanding and Forecasting Business Cycles", *Melbourne Institute Working Paper*, No. 17/01.

Chauvet, M. (1998), “An Econometric Characterization of Business Cycle Dynamics with Factor Structure and Regime Switching”, *International economic review*, Vol 39 (4), 969-996.

Diebold F. and Rudebusch G. (1996), “Measuring Business Cycles: A Modern Perspective”, *Review of Economic and Statistics*, 78, 67-77.

Hamilton J. (1989), “A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle” , *Econometrica* , 57 (2), 357-384.

Hamilton J. (1990), “Analysis of Time Series Subject to Change in Regime”, *J.o.Econometrics*, Vol 45, 39-70.

Harvey A. (1989), “Forecasting, Structural Time Series Models and the Kalman Filter.” Cambridge: Cambridge University Press.

Kim, C.J and Nelson C.R. (1998), “Business Cycle Turning Points, a New Coincident Index, and Tests of Duration Dependence Based on a Dynamic Factor Model with Regime Switching”, *Review of Economics and Statistics*, 80(2), 188-201.

Kim C.J. and Nelson C.R. (1999), “State-Space Models with Regime Switching.” Cambridge, Mass.

Melnick R. and Golan Y. (1992), “Measurement of Business fluctuations in Israel”, *Bank of Israel Economic Review*, No.67, 1-20.

Melnick R. (1992), “Forecasting Short-Run Business Fluctuations in Israel” (Hebrew), *Discussion Papers Series*, No 17, Bank of Israel,.

Melnick R. (2002), “Business Cycles in Israel", The Economic Quarterly, 219-244 (Hebrew).

Moore G. H. (1983) “Business Cycles, Inflation and Forecasting”, 2-ed., *NBER Studies in Business Cycles*, No 24 , Cambridge, Mass.

Smith, P. and Summers P. (2001), “ Synchronization of Business Cycles in the G7 Countries: Evidence from Bayesian Analysis of Markov Switching Models “, Mimeo, The University of Melbourne.

Smith, P. and Summers P. (2002), “ New Coincident Indicators for Australia’s Major Trading Partners”, *Working Paper Melbourne Institute,* The University of Melbourne.

Stock, J.H. and Watson M.W. (1989), “A probability model of the coincident economic indicators “, in *NBER Macroeconomics Annual*, ed. O.Blanchard and S.Fischer, Cambridge, Mass, 351-394.

Stock, J.H. and Watson M.W. (1993), “A Procedure for Predicting Recessions with Leading Indicators: Econometric Issues and Recent Experience" , in *Business cycles, indicators and forecasting*, ed. J. H.Stock and M.W.Watson, Chicago, 95-156.

Zarnowitz V. and Ozyildirim A. (2002) “Time Series decomposition and measurement of Business cycles, Trends and Growth Cycles”, in *NBER Working Paper Series*, No 8736.

Appendices

A. The components weights of the coincident indices in different countries[11]

AUSTRALIA

1. Retail Trade	.0822
2. Unemployment Rate	.4612
3. Industrial Production	.0468
4. Employed Persons	.2828
5. Household Disposable Income	.1270

USA

1.Employees on nonagricultural Payrolls	.5230
2.Personal income less transfer payments	.2176
3. Industrial production	.1407
4. Manufacturing and trade sales	.1187

UK

1 Industrial Production	.0571
2 Retail Sales	.0548
3 Employment	.7402
4 Real Household Disposable Income	.1479

FRANCE

1. Retail sales	.0940
2. Industrial Production	.0531
3. Real Imports	.0532
4. Paid Employment	.7997

SPAIN

1. Final Household Consumption	.6215
2. Industrial Production, excl. Construction	.2477
3. Retail Sales Survey	.0611
4. Real Imports	.0697

KOREA

1. Industrial Production	.0428
2. Wholesale and Retail Trade	.0534
3. Employment	.1966
4. Unemployment Rate	.7072

MEXICO

1. Industrial Production	.1086
2. Retail Sales	.0504
3. Employment	.1663
4. Unemployment Rate	.6747

GERMANY

1. Manufacturing Sales	.0591
2. Industrial Production	.1018
3. Retail sales	.0963
4. Employment	.7428

---

[11] Source: The Conference Board in *http://www.tcb-indicators.org/*

B. The State-Space Representation of the model

$Y_t = HZ_t + \varepsilon_t$ - measurement equation

$Z_t = \mu + FZ_{t-1} + \eta_t$ - transition equation

where Y_t (vector of index components), Z_t (state vector) and η_t (vector of residuals) look as follows:

$$Y_t = \begin{pmatrix} i_t \\ m_t \\ r_t \\ e_t \\ ex_t \end{pmatrix}, Z_t = \begin{pmatrix} \Delta C_t \\ \Delta C_{t-1} \\ \Delta C_{t-2} \\ \Delta C_{t-3} \\ \varepsilon_t^{(i)} \\ \varepsilon_{t-1}^{(i)} \\ \varepsilon_t^{(m)} \\ \varepsilon_{t-1}^{(m)} \\ \varepsilon_t^{(r)} \\ \varepsilon_{t-1}^{(r)} \\ \varepsilon_t^{(e)} \\ \varepsilon_{t-1}^{(e)} \\ \varepsilon_t^{(ex)} \\ \varepsilon_{t-1}^{(ex)} \end{pmatrix}, \eta_t = \begin{pmatrix} \upsilon_t \\ 0 \\ 0 \\ 0 \\ \varepsilon_t^{(i)} \\ 0 \\ \varepsilon_t^{(m)} \\ 0 \\ \varepsilon_t^{(r)} \\ 0 \\ \varepsilon_t^{(e)} \\ 0 \\ \varepsilon_t^{(ex)} \\ \end{pmatrix}, \quad \begin{aligned} \nu_t &= i.i.dN(0, \sigma_{\Delta C}^2) \\ \varepsilon_t^{(i)} &= i.i.dN(0, \sigma_{(i)}^2) \\ \varepsilon_t^{(m)} &= i.i.dN(0, \sigma_{(m)}^2) \\ \varepsilon_t^{(r)} &= i.i.dN(0, \sigma_{(r)}^2) \\ \varepsilon_t^{(e)} &= i.i.dN(0, \sigma_{(e)}^2) \\ \varepsilon_t^{(ex)} &= i.i.dN(0, \sigma_{(ex)}^2) \end{aligned}$$

when the intercept vector μ and system matrices *H* and *F* look as follows:

$$H = \begin{pmatrix} \gamma_0^{(i)} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma_0^{(m)} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma_0^{(r)} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma_0^{(e)} & \gamma_1^{(e)} & \gamma_2^{(e)} & \gamma_3^{(e)} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma_0^{(ex)} & \gamma_1^{(ex)} & \gamma_2^{(ex)} & \gamma_3^{(ex)} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mu = \begin{pmatrix} \mu_{S_t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad F = \begin{pmatrix} \varphi_2 & \varphi & \varphi_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi_1^{(i)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \psi_1^{(m)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_1^{(r)} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_1^{(e)} & \psi_2^{(e)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_1^{(ex)} & \psi_2^{(ex)} \end{pmatrix}$$

C. Parameter estimates.

Variables	Parameters	Estimates	
ΔC_t	q	0.965	
	p	0.777	
	μ_0	-0.005	
	μ_1	0.006	
	φ_1	-0.3	(-7.9)
	φ_2	0.04	(1.1)
	φ_3	0.78	(21.5)
i_t	$\gamma_1^{(i)}$	1.18	(25.6)
	$\psi_1^{(i)}$	-0.33	(-9.8)
m_t	$\gamma_1^{(m)}$	2.01	(15.1)
	$\psi_1^{(m)}$	-0.62	(-16.3)
r_t	$\gamma_1^{(r)}$	1.85	(18.5)
	$\psi_1^{(r)}$	-0.79	(-23.4)
e_t	$\gamma_1^{(e)}$	4.71	(42.5)
	$\gamma_2^{(e)}$	0.75	(13.4)
	$\gamma_3^{(e)}$	-0.16	(-3.2)
	$\gamma_4^{(e)}$	-3.28	(-31.5)
	$\psi_1^{(e)}$	0.24	(6.2)
	$\psi_2^{(e)}$	0.57	(16.0)
ex_t	$\gamma_1^{(ex)}$	2.53	(3.6)
	$\gamma_2^{(ex)}$	0.05	(0.2)
	$\gamma_3^{(ex)}$	-0.62	(-2.0)
	$\gamma_4^{(ex)}$	-1.11	(-1.8)
	$\psi_1^{(ex)}$	-0.67	(-14.9)
	$\psi_2^{(ex)}$	-0.37	(-5.5)
Log likelihood	-1839.2		

Note: t-statistics are in parentheses.