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PREDICTING GOVERNMENT TAX REVENUES AND ANALYZING FORECAST UNCERTAINTY

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Abstract

This study examines the sources of uncertainty in predicting government tax revenues in Israel. In the first stage, we estimate a model based on several real and financial macroeconomic variables and identify a significant, stable and highly accurate relation between these variables and tax receipts. Moreover, we find that, given these variables, current tax revenues do not improve the projection of revenues. In the second stage, we test the quality of the model's projections on the basis of available information at the time the budget is prepared; we find that the forecast error based is six times greater than the error based on ex-post projection. These results imply that the forecast error predominantly reflects inaccuracy in the prediction of the explanatory variables and not misidentification of the relations among the variables. In particular, we find that GDP projections tend to be overly pessimistic—especially when they are prepared at times of below-average growth. In the third stage, we ask whether limited versions of the model predict tax revenues better; we find that the removal of the financial variables and the indicator for new-dwelling sales does improve the projection. However, models that are even more limited—based only on lagged tax revenues and a GDP growth forecast—provide less-accurate projections, and the probability that they will lead to significant errors in the construction of the budget is greater than that of the broader models.

1. INTRODUCTION

The tax-revenue forecast is an essential component of short- and medium-term fiscal policy. In a regime of deficit targets, below-forecast revenues may increase the deficit, undermine public confidence in fiscal policy, and force the government to revise its budget in mid-year in order to attain its deficit target—a situation in which it cannot always choose the most efficient adjustments. Projection that turns out to be too low may also cause

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damage, by ruling out activities of public value or causing some of the public to suspect the government of manipulating the projection in order to avoid various expenditures or tax cuts. Hence, accurate revenue projections provide important benefits. Projections, however, are always uncertain to some extent, due to incomplete information about the relations between revenue levels and the variables used in producing the projection, and also due to incomplete information about the values of these variables. Therefore, it is also important to understand and quantify the inherent risks of the projection. Estimating the probability of errors of various magnitudes in the forecast[1] may help the government to make informed decisions about the risks that it is willing to take in regard to the budget at a given time in accordance with circumstances. For example, when mulling policy measures that entail considerable budget expenditure in coming years, obtaining an estimate of the probability that revenues in these years will be significantly lower than the present forecast may allow the government to decide whether such risks are acceptable or whether the expenditure should be postponed. By presenting the risks within the framework of the public discourse, too, the government may enhance its credibility—and its responsibility—if the risks come to pass.

Until recent years, the projections that the Israeli Ministry of Finance presented to the public with the budget were based on simplistic models that assumed unit elasticity of government tax revenues with respect to nominal GDP, adjusted for the effect of legislative changes in tax rates. In some years the projections also included estimates for the effect of efforts to "intensify tax collection" (*Main Provisions of the Budget*, 1992–2000). In recent years, the analysis frame has changed somewhat; it also includes reference to the effect of changes in imports and wages (mainly on the composition of tax revenues and less on total revenues). However, the projection is still based mainly on the proposition that revenues will increase at a rate resembling that of GDP. As for actual revenues, the budget forecasts have occasionally been "off" by rather large amounts in recent years—in both directions; the annual reports of the State Revenue Administration provide ex-post explanations for the deviations.

In the present study, we test an econometric model for the prediction of tax revenues. A model of this type has an advantage over current practice in that it better reflects the economic relations that influence the behavior of tax revenues. It also, however, has the drawback of being at least partly dependent on predictions of the values of the explanatory variables. Thus, we examine two kinds of errors—an error of the model and an error in predicting the explanatory variables—and ask which of them is the greater. On this basis, we then ask whether broader models have an advantage over the aforementioned simple model even when the prediction error in the explanatory variables is taken into account. The analysis in this article, like the State Revenue Administration's *ex post* analyses, focuses on the effects of macroeconomic variables on the revenue forecast and does not concern itself with predicting the effects of legislative changes and administrative measures (see discussion below).

An example of a broad model of the type we examine is the total government tax revenue function estimated by Brender (2001). This function is based on a reduced form of

[1] For example, a 25 percent likelihood that revenues will be NIS 2 billion below the projection.

empirical relations between real and financial macroeconomic variables and revenues, without identification of structural relations.[2] The model estimated manages to track the factors that affect revenues with great accuracy, even at points of time at which the GDP growth rate and the revenue trend changed considerably, and its coefficients are stable across the period (Bank of Israel *Annual Report*, 2005). According to that model, the rate of change in tax revenues in the long term is only slightly higher than that of GDP, while in the short term it may be much larger, especially at times of change in the growth rate. Therefore, relying on the long-term relation of revenues with GDP for short-term projections may lead to considerable errors. This finding supports the use of broad models because such errors acquire immense importance in policymaking: Israel's budget framework and binding targets are mainly short-term, i.e., annual, for the current year to which the revenue projection relates. Long-term analyses usually serve only as background material for the budget deliberations.

Although Brender's model **describes** the development of tax revenues well, the question is whether it is an effective instrument for the **prediction** of these revenues. The reason is that the revenue projection (net of legislative changes) includes two types of errors: (1) a forecast error by the model itself (2) incomplete information and error in predicting the explanatory variables of the model. Since these variables are usually unknown at the time the forecast is prepared, a more accurate profile of the forecast risks may be obtained if one takes the uncertainty of their values and their covariance into account. In *ex post* analyses of revenues—such as that in Brender (2001)—the second component is of limited importance; it is, however, an important component in a projection (as the aforementioned article does note).

In the past decade, the macroeconomic and financial literature has placed growing emphasis on the uncertainty in forecasts (Diebold, 1995, Diebold and Lopez, 1996; Clements, 1998, 1999; Granger and Pesaran, 2000a, 2000b). Whereas in the past the application of models focused mainly on point estimates of the predicted variables, the frame of reference has expanded in recent years to include analysis of the distribution of expected values of the dependent variable and the inherent risk of the forecast. The best known example of this use is the Bank of England's "fan chart", which presents the forecast generated by the bank's inflation model not only in terms of point estimate but also in terms of the distribution of the error probabilities of the forecast in various orders of magnitude (Wallis, 2003, 2004; Clements, 2004; Cogley et al., 2003). The presentation is made in this fashion due to the realization that it is important to understand the risks to which the user of the forecast is exposed, since the "price" of deviations of various magnitudes and in various directions is not necessarily symmetrical (Levin et al., 2005; Ashley et al., 2005). This rule also applies to the tax-revenue projection. At times of global economic shocks, for example, the government may believe that a 20 percent probability of a 0.5 percent of GDP shortfall in tax revenues relative to the projection reflects too great a risk and will prefer to take revenue-boosting measures, whereas at times of economic calm it may consider such a risk reasonable. In another example, if large deviations may be much costlier than small errors, policymakers will prefer to reduce the probability of such errors even if the budget

[2] For the sake of brevity, below we term the functions estimated in this manner a "model."

targets are consistent with the point estimates. Although Celasun et al. (2006) carried out initial studies on estimating the fan of fiscal projections, this field is still in its infancy and has not yet been applied in Israel. Fan-like tools have also been used to analyze financial risks in the business sector (Diebold, 1998; Berkowitz, 2001).

The present study has three parts: (1) estimation of the tax model in order to identify the factors that affect the development of government tax revenue in the short term; (2) analysis of the relative contributions of the model coefficients and of uncertainty in predicting the explanatory variables to the quality of the model's forecasts; (3) analysis of forecast quality provided by different versions of the model, by means of various tests that appear in the recent literature (KLIC tests). We do this in order to examine the tradeoff between the added explanatory power attained by improving the model and the loss of accuracy resulting from uncertainty in predicting the additional explanatory variables (Berkowitz, 2001; Wallis, 2004; Mitchell and Hall, 2005; Sarno and Valente, 2004). Section B presents the real and financial macroeconomic variables that will be tested in the model and Section C shows the results of the estimation of the model and indicators of its success in explaining the relation between tax revenues and the explanatory variables, and stability of the model. Section D presents the available information about the explanatory variables at the time the forecast is prepared. Section E tests the quality of the forecast given this information and proposes alternative models that take account of the uncertainty about the values of the explanatory variables. Section F calculates the uncertainty of the projection elicited by the model and Section G concludes and suggests future directions of research.

2. VARIABLES AFFECTING TAX REVENUES

a. The dependent variable

Tax revenues depend on several macroeconomic and financial variables, some concurrent and others lagged. As our point of departure, the model bases itself on variables that reflect the main tax bases: GDP, imports, the housing market, and financial-sector activity (Brender, 2001). The main tax bases and the tax revenues directly related to them are shown in Table 1. The exact specification of the model—particularly the lags chosen—reflects the results of empirical estimation and is not a choice based on a structural analytical model. For this reason, the coefficients of some variables may reflect the correlation between them and other tax bases and not only the direct effect on revenues of changes in the tax base that the variable ostensibly represents. In the discussion below, we describe the variables chosen for the model and explain the rationale for choosing them. We also mention several variables that were omitted from the final version because their contribution to the model was not significant.

The dependent variable in all equations is **central government tax revenue net of the effect of legislative changes.** Reasonably good quarterly data on government tax revenues have been available in Israel since the early 1990s; they allow us to estimate a quarterly revenue model including income-tax receipts (net of refunds), real-estate taxes, net Value

Added Tax, import taxes, indirect taxes on domestic production, fees, and other compulsory payments.[3] Social insurance contributions were omitted from the estimate because they affect on the central government budget deficit—the official target of fiscal policy—less than the other taxes,[4] and the national health insurance tax was omitted because it was not collected until 1995 and it is not recorded in the budget. All financial data in this study are in constant prices.[5]

Table 1
Tax Bases and Corresponding Receipts: 2006 (NIS millions)

	Tax base	Tax revenue
GDP	633,057	178,910
Total wage bill in the economy	309,759	43,900
Imports of consumption goods	26,211	18,923[1]
Sales of new dwellings	20,995	4,885[2]
Credit in and indexed to foreign currency	108,851	2,041[3]
Israeli Stock Issues abroad[4]	65	—

[1] Sales tax on imports, customs, and imputation of VAT on imports of consumption goods.
[2] Excluding VAT but including real-estate purchase tax on existing dwellings.
[3] Assuming 7.5 percent average interest and a 25 percent average tax rate.
[4] 2004–2006 average.

Since this study focuses on the direct relation between macroeconomic variables and revenues,[6] we subtract from the revenue data the effect of legislative changes each year in accordance with estimates presented in the State Revenue Administration's reports in the year following the change. Legislative changes had to be excluded from the measurement of tax data because several significant tax-related measures were taken during the estimation period. They included the gradual lowering of tariffs and purchase-tax rates on imported goods as part of the trade liberalization, the gradual lowering of corporate tax rates between 1991 and 1996, income-tax reforms since 2004, and VAT reductions in 2005 and 2006. Estimating the effect of legislative changes on revenues is important in itself, but most year-on-year changes in revenues reflect the effect of macroeconomic developments on the tax base and not that of legislative changes. Between 1993 and 2006, the effect of legislative changes added up to 1.4 percent of total revenues on annual average (year on

[3] The revenues used in this study are based on actual collection and not on assessments presented to taxpayers, as is conventional in empirical studies on public economics in the literature. Insofar as the difference between the two metrics correlates with the explanatory variables, this may create a bias. However, apart from the isolated case of shifting tax refunds for several days in 2004, there is no evidence of such a systematic relation in refunds or in collection and enforcement efforts.

[4] In some areas of National Insurance (social insurance) activity, government transfers the National Insurance Institute are a function of collection from the public. Therefore, when the collection of National Insurance contributions increases, so does central government expenditure and, in turn, the budget deficit.

[5] All variables were calculated in nominal values and deflated by the Consumer Price Index.

[6] As Brender (2001) shows, legislative changes are pro-cyclical. However, this matter is beyond the scope of this article.

year in absolute values) whereas the change in revenue net of the effect of legislative changes was 9.7 percent.[7]

Given Brender's (2001) finding that estimating separate equations for the different types of taxes would make only a minor contribution to the explanation of total revenue, this study focuses only on the total revenue equation and not on separate equations for individual taxes. We did attempt to estimate separate equations, but again they did little to improve the total explanatory power and yielded less stable coefficients. Furthermore, specific-tax equations necessitate more data for the projection, most of which are not available at the time the projection is prepared; therefore, they expand the range of potential errors in the projection.

b. Real variables

Gross Domestic Product. The main tax base, which reflects overall economic activity and the added value generated in it. The effect of GDP is not necessarily concurrent because some tax payments are made on account of lagged activity. For this reason, GDP is included with a lag of one-quarter.[8] The use of GDP does more to explain revenue than business-sector output. The variable of private consumption did not enhance the explanatory power of the equations.

Change in GDP (growth): decelerations and accelerations of growth have a short-term effect on tax revenues beyond the effect of the level of GDP. This is because some tax remittances by firms and the self-employed are based on advance payments, the level of which is adjusted at a lag to their volume of activity, and on the payment of differentials that are affected by their level of earnings. Earnings themselves vary in a way that is disproportional to changes in growth. Furthermore, at times of faster growth, imports of intermediates and capital goods sometimes increase, delaying the increase in net collection of Value Added Tax. As a result, revenue increases disproportionately at times activity accelerates and decreases disproportionately when activity decelerates.

Real wage per employee post: since wage taxes are progressive and because the marginal tax rate on labor income is higher than the effective tax rate on capital income, the distribution of added value between wages and capital has a perceptible effect on total tax revenues. To isolate the marginal contribution of the wage variable—which correlates with GDP—we included in the equations the deviation of wages from the level we would predict

[7] The effect of legislative changes was netted out in the following way: (a) the data on the effect of the legislative changes on tax revenues were culled from the annual report of the State Revenue Administration; (b) the change in revenues was divided by total tax revenues in the relevant year in order to obtain the percent change in tax revenues as a result of the legislative changes; (c) the revenue data for subsequent years were adjusted by the same percent to offset the effect of the legislation. The change in tax revenues was recorded in the year when the legislative change affected collection and not in the year of the legislation (State Revenue Administration, 2006). The baseline year for the estimation of tax revenue net of legislative changes is 1998.

[8] The GDP data and all other data used in the estimation are the most recent data publicly available at the present writing, and not the initial estimates published by the Central Bureau of Statistics. These data are based on the full set of information gathered by CBS over the years for these periods of time.

for it on the basis of its long-term relation with GDP.[9] This tax-equation specification does not change the equation's coefficient of explanation; it affects only the coefficients of the wage and GDP variables; thus the wage coefficient reflects only the contribution of the change in wage that is beyond the relation between wage and GDP. The alternative variable of total wage payments in the economy (the product of the number of employees and the wage per employee post) has less explanatory power, evidently because changes in employment are mainly reflections of the entry and exiting of workers in low tax brackets.[10] Changes in the average real wage, in contrast, reflect wage developments among all workers, whose average marginal tax rate is higher. The wage variable included in the tax-revenue equation is unlagged because wage taxes are usually paid through direct withholding by employers.

Imports of consumption goods: imported consumption goods are not part of GDP; they are an additional tax base that is taxed at especially high rates (Table 1). The taxes applied to this base (including VAT) are imposed when the goods enter the country; since this is also when the imports are recorded in the foreign-trade data, this variable is contemporaneous in the equation. It is calculated on the basis of its domestic-currency value in constant prices. Since this variable is strongly correlated with GDP, we included in the tax equation its deviation from its long-term relation with GDP and not its level, as with the wage variable. Given the large changes in the tax-rates on consumption-goods imports since the end of 2000, we examine whether the effect of imports on tax revenues changed from 2001 onward. To answer this question, the estimation includes, apart from the deviation of imports from the long-term relation, an interaction variable (a slope dummy) of deviation of imports with a binary variable that receives the value of zero for 1991–2000 and the value of 1 for 2001 and subsequent years.

Sales of new dwellings: residential construction is recorded in GDP on a current basis in accordance with progress in construction, while much of the tax on account of dwelling purchase is paid after the purchase—with no reconciliation performed with the recording of the construction in GDP. Therefore, our equation includes new-dwelling sale transactions as an additional explanatory variable. The attempt to include total housing sales transactions (including existing homes and owner-built homes) or the sales of existing dwellings in lieu of, or in addition to, this variable (respectively) did not enhance the explanatory power of the equation. Three factors evidently explain why sales transactions for existing dwellings did not contribute significantly to the explanatory power: the quantitative importance of VAT payments, which apply only to new-dwelling sales; the exemption from land-revaluation tax that applied to most existing-home sales transactions throughout the research period, and the strong correlation between sales of dwellings of the two types. Given the lapse of time between the sale and the payment of taxes, the variable was included at a three-quarter lag. (In the differentials equation, it is included at a two-quarter lag.)

[9] This variable was estimated as the residual of an Engle-Granger cointegration regression of wages on GDP; the GDP variable included in this estimation is lagged by exactly the lag included in the tax equation.

[10] Evidence for this appears in Brender and Strawczynski (2006), who find that the probability of entering and leaving employment is much greater among part-time workers, most of whom are below the tax threshold or in low tax brackets.

c. Financial variables

In addition to real activity, taxes are imposed on financial income. Furthermore, some developments in the field of finance affect, or correlate with, corporate profitability and real activity. For this reason, we also examined the relation between several financial variables and government tax revenues. In addition, we asked whether one can identify an effect of legislative changes that broadened the taxation of interest earnings and capital gains of the public at large from 2004 on, but we were unable to detect such a relation.[11] This may be due to the small share of this item in total tax revenue when it was first implemented: less than 1 percent of total revenue in 2004 and 1.3 percent in 2005. Furthermore, much of the increase in revenue on this account reflected the gradual broadening of the tax base in accordance with the original legislation and not changes in the earnings themselves.[12]

Mergers and sale of shares abroad by parties of interest (defined henceforth as those holding at least 5% of the company's shares): merger transactions with foreign firms and issues of shares by parties of interest abroad are an indicator of Israelis' capital gains in transactions with nonresidents. The tax liability for these transactions is not immediate; indeed, we found that the strongest contribution of this variable to explaining tax revenues appears at a lag of two or three quarters—a finding consistent with the procedure for actual payments of tax on account of these transactions. This variable evidently also reflects the effects of domestic capital-market activity on revenues, since its inclusion in the equation excludes from it the change in prices of the shares on the Tel Aviv Stock Exchange. Furthermore, it does more to explain revenues than the alternative variable: issue of Israeli shares abroad.

Bank credit in or indexed to foreign currency: the amount of credit denominated in or indexed to foreign currency, taken by the business sector, increased perceptibly during the 1990s and until 2003 and began to decline steeply in 2005. Interest payments on this credit, as on any other credit, are a deductible expense for borrowers whereas most income on this interest is not taxable in Israel because the sources for the loans come from abroad (even if the loans are administered by Israeli banks, as is often the case); therefore, one would expect this variable to have an adverse effect on tax revenues. Banks' income from interest on lending in domestic currency, in contrast, originates in domestic sources (the public or the Bank of Israel) and entails a much smaller reserve ratio than that for the issue of foreign-currency loans. (The contribution of the "total credit in domestic currency" and "domestic-currency interest" variables to the explanation of revenues was not significant.) The foreign-currency credit variable was included at a lag that reflects the time interval between the increase in interest expenses (e.g., due to currency depreciation) and the reckoning vis-à-vis the income-tax authorities that takes place in order to make these expenses deductible.

[11] Capital-market professional dealers and holders of significant shares in traded corporations were liable to withholding at source on their income before 2004 as well.

[12] Financial assets that were created/acquired before the law was applied were exempt from tax until first payback.

Change in the TA General Share Index: capital gains, including parties of interest realized capital gains from the sale of shares, are not included in GDP but are liable to income tax. Since 2004, capital-gains tax has been applied to the public at large. To estimate these gains and the extent of their realization, we used the average real change in the General TA Share Index (in points) in the quarter. Although we used various profiles of this variable in our efforts to estimate the equations, in most specifications it was significant and in others the coefficient and the significance were not robust to changes in the equation specification.

d. Characteristics of the variables' time series

The averages and standard deviations of the variables are shown in Table 2. An examination of the degree of cointegration of the variables shows that tax revenues, GDP, wage, imports of consumption goods, and foreign-currency credit are typified by an I(1)

Table 2
Descriptive Statistics of Variables in the Model

	Average	S.D.
Log tax revenues less legislative changes	10.32	0.19
Log GDP	11.56	0.17
Deviation of log wage from its long-term relation with GDP	0.0	0.03
Deviation of log imports from its long-term relation with GDP	0.0	0.08
Deviation of log imports from relation with GDP (since 2001)	0.0	0.04
Mergers and sales of shares abroad	150.5	296.6
Log credit in and indexed to foreign currency	11.14	0.62
Change in log GDP (growth)	0.01	0.02
Log new-dwelling sales	9.04	0.24

stochastic process (Table 3). The **change** in GDP, the deviation of imports and wages from their relation with GDP, new-dwelling sales, and mergers and sales of shares abroad by parties of interest, in contrast, are typified by an I(0) process. In estimating the model, these characteristics of the series should be taken into account. The finding that GDP is typified by an I(1) stochastic process, with no trend, is different from the outcome that one would expect according to Hayashi's analysis (Hayashi, 2000, pp. 557–622), i.e., a stationary variable with a trend. However, similar findings were found in previous works on the Israeli economy (Yakhin and Menashe, 2001; Lavi and Strawczynski, 2001). In Appendix 1 we test the possibility that the Israeli GDP series is stationary with a trend both in annual data for 1973 to 2007 and in quarterly data from 1990 to 2010, but we find no support for it (Appendix A, Tables A-1 and A-2).[13]

[13] Since the series for Israel are rather short, one cannot totally dismiss the possibility that GDP is in fact stationary with a trend. For this reason, we also tried to add a time trend to the model that appears in Table 6. The time-trend variable was not significant (P=0.85) and had no qualitative effect on the other coefficients.

Table 3
Degree of Cointegration of Variables in Total Tax-Revenue Equation [1]

	ADF test[2] for level I(0)	ADF test[2] for change in variable (first difference)
Tax revenues	-1.986	-7.199*
GDP	-1.850	-5.258*
Change in GDP (growth)	-5.258*	-9.797*
Real wage per employee post	-0.924	-6.322*
Imports of consumption goods	-2.071	-7.508*
New-dwelling sales	-6.047*	-7.991*
Mergers and sales of shares abroad	-4.078*	-6.635*
Credit in and indexed to foreign currency	-2.653	-3.271**
Deviation of imports from its relation with GDP	-3.076*	-8.204*
Deviation of wages from its relation with GDP	-4.549*	-8.178*

[1] All the variables, except "mergers and sales" are in logs.
[2] Estimated with intercept and two lags.
* Significant at 1% level.. ** Significant at 5% level.

Apart from the characteristics of the time series of the variables, the estimation should take account of seasonality in tax collection, resulting from the payment of some taxes on regular dates, e.g., bimonthly VAT reports. Since firms are not distributed equally among dates of payment, revenue size in even-numbered months is different from that in odd-numbered months. To reflect this and other seasonal effects, we include quarterly dummy variables.

3. RESULTS

To test the relation of the macroeconomic variables with tax revenues, we estimate a cointegration model using the Engel-Granger method (1987). The long-term equation tests the relation of quarterly tax revenues with GDP (in log), with quarterly dummy variables added, in 1991–2006 (Table 4). The equation shows that in the long term (insofar as the estimation period may be considered such), tax-revenue elasticity to GDP, net of legislative changes, is 1.08 (different from 1 at 5 the percent significance level—t-statistic 2.26), similar to the elasticity found by similar studies in developed countries (Van Den Noord, 2000). The residuals of this equation are stationary and the statistic of the Augmented Dickey-Fuller (ADF) test for the residuals of the long-term equation is (–4.19)—significant at a 1 percent level.

The short-term equation (the first-differences equation) asks whether the **changes** in tax revenues correlate with **changes** in GDP and in the aforementioned macroeconomic variables (Table 5). This equation also includes the residuals of the long-term equation at a one-quarter lag. In other words, we ask whether, when revenues deviate from the expected outcome given the relation with GDP, revenues converge to the expected level in subsequent quarters (error correction).

Table 4
"Long-Term" Equation for Estimation of Total Tax Receipts Net of Effect of Legislative Changes[1]

Intercept	–2.220
	(–5.25)
Log GDP	1.082
	(29.80)*
Dummy for 1st Quarter	0.076
	(4.78)*
Dummy for 3rd Quarter	0.037
	(3.07)*
Adjusted R-squared	0.951
Durbin-Watson stat	0.74
F-statistic	386.25
ADF statistic of residuals	–4.19*

[1] The estimation is for the Q1:1991-Q4:2006 period.
The numbers in parentheses are t-statistics.
[*] Statistical significance at 1% level.

Table 5
Change in Tax Revenue Equation

	First Differences equation[1]	
	Coefficient	t statistic
Intercept	-0.506	(-3.30)*
Log GDP at one-quarter lag	1.051	(5.24)*
Change in log GDP in past 3 quarters	0.477	(3.76)*
Log wage deviation from its relation with GDP	0.505	(3.46)*
log consumption-goods imports deviation from its relation with GDP	0.250	(6.17)*
Log foreign-currency credit (4-qtr. Lag)	-0.179	(-2.75)*
Log new-dwelling sales (2-qtr. Lag)	0.051	(3.02)*
Mergers and sales of shares abroad (2-qtr. Lag)	0.028	(3.02)*
Mergers and sales of shares abroad (3-qtr. Lag)	0.024	(3.17)*
Dummy for 1st Quarter	0.128	(11.67)*
Dummy for 3rd Quarter	0.054	(4.56)*
Residual of long-term equation (1-qtr. Lag)	-0.344	(-5.01)*
Adjusted R-squared	0.89	
Durbin-Watson stat	2.35	
F-statistic	45.13	
ADF statistic of residuals	-9.33*	

[1] The estimation is for Q2:1991–Q4:2006. All variables are calculated in terms of change relative to previous period. The numbers in parentheses are t statistics.
[*] Significance at 1% level.

The equation shows that a cointegration relation between taxes and GDP does exist: the residuals of the "long-term" equation have a significant and negative effect on the change in revenues.[14] In other words, when revenues exceed the level that corresponds to their relation with GDP, the pace of their increase in subsequent periods slows. The short-term adjustment coefficient is –0.344, i.e., about half of the deviation is corrected within two quarters and 80 percent is corrected within a year. We also find that **changes** in tax revenues are positively correlated with **changes** in GDP, the change in GDP appearing at a one-quarter lag and the elasticity slightly above one. Additionally, changes in the growth rate make an additional contribution to the explanation of the change in revenues, with a 0.4 elasticity. Furthermore, changes in the deviations of wages and imports from their long-term relation with GDP, new-dwelling sales, and mergers and sales of Israeli shares abroad by parties of interest have a positive effect on revenues. Changes in foreign-currency credit have a negative effect on revenues, as expected.

An alternative way to estimate the relations is by using a single equation that includes variables that reflect the long-term relation of tax revenues with macroeconomic developments along with the variables that reflect changes in short-term revenues. Such an equation appears in Table 6.[15] Estimating this equation, we found that the residuals are

Table 6
Total Tax Revenues Net of Effect of Legislative Changes Equation[1]

	Coefficient	t-statistic
Intercept	-4.168	-5.84*
Lagged log GDP	1.276	15.54*
Lagged log GDP * dummy variable for after-2000	-0.003	-3.10*
Change in growth rate (2-qtr. Lag)	0.342	2.92*
Deviation of log wage from relation with GDP	0.526	4.53*
Deviation of log consumption-goods imports from relation with GDP	0.477	9.70*
Deviation of log consumption-goods imports from relation with GDP * dummy variable for after-2000	-0.184	-2.73*
Mergers and sales of shares abroad (2-qtr. Lag)	0.017	2.03**
Log foreign-currency credit (1-qtr. Lag)	-0.041	-2.05**
Dummy for 2nd Quarter	-0.019	-2.08**
Dummy for 4th Quarter	-0.067	-8.14*
Log new-dwelling sales (3-qtr. Lag)	0.024	1.43
Adjusted R-squared	0.99	
S.E. of regression	0.02	
Durbin-Watson stat	1.97	
F-statistic	355.76	
ADF statistic of residuals	-7.74	

[1] Total tax revenues net of the effect of legislative changes, according to Tax Division data in 2000 prices.
* Significant at 1% level. ** Significant at 5% level

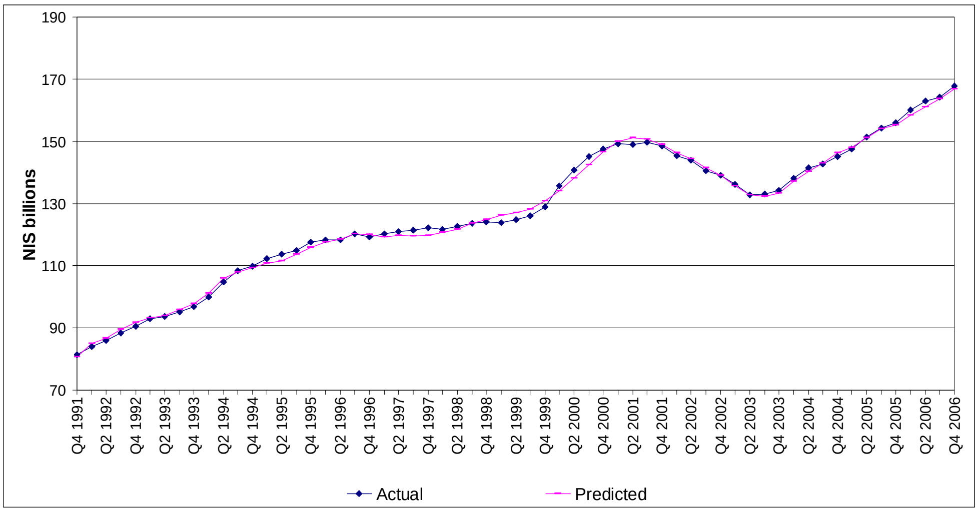
[14] As stated above, in the ADF test we found that the residues are I(0).
[15] The R^2 of the equation should not be treated with much importance because the dependent variable and several explanatory variables exhibit an I(1) stochastic process, as stated.

indeed stationary, as required; that tax-revenue elasticity to GDP is greater than one; and that changes in the growth rate have a positive, significant, and strong effect on tax revenues. Thus, a 1 percentage-point increase in the growth rate boosts revenues by one-third of a percentage point. This factor explains the substantially greater-than-unit elasticity observed at times of change in GDP growth and reflects the reversion to the "normal" level of revenues when growth rates level off.

The development of wages correlates positively with tax revenues, as expected. When the national average wage rises by more than its relation with GDP explains, revenues rise with an additional 0.5 elasticity. Similarly, an increase in imports of consumption goods at a rate exceeding its long-term relation with GDP boosted revenues with 0.5 elasticity until 2001. After 2001, this elasticity decreased by nearly 40 percent—to 0.3, in tandem with the decrease in import-tax rates. New-dwelling sales have a favorable, but small, effect on revenues and at a level of significance that is not robust to changes in the equation's specification. Parties of interest sales of shares abroad affect revenues positively and foreign-currency credit affects revenues negatively. Adding the dependent variable at a lag did not improve the explanatory power of the equations and did not exclude any of the explanatory variables.

The quality of the forecast obtained in a resolution that is relevant for decision-making—one year—is shown in Figure 1, which compares the four-quarter moving average of the tax-revenue projection elicited by the equation with actual revenues. The paths prove to be very similar, including at major turning points in the past fifteen years. Even if perceptible differences between the model projection and actual revenues sometimes occur at the quarterly level, they are quickly offset.

Figure 1
Tax-Revenue Projection and Actual Revenues, 1991–2006
(Four-quarter moving average)



An alternative way to examine the success and stability of the model in describing the relation of tax revenues with the explanatory variables is by testing the accuracy of the forecasts that the model yields. Table 7 presents the projections for 2000–2006 as derived each year from the model that was estimated to the end of the previous year. In the equations for each year, we inserted the actual values of the explanatory variables in the forecast year. The table shows that the forecast error was very small: the absolute error did not exceed NIS 1.4 billion (1 percent of revenues in 2004) and the average absolute deviation was NIS 738 million (0.5 percent of revenues or 0.15 percent of GDP). These findings also show that there is little to gain from reducing the revenue projection to its components—individual taxes—because this kind of estimation does not add much to the explanatory power.

4. USING THE MODEL TO PREDICT TAX REVENUES

The model presented above, as stated, describes the relations between macroeconomic variables and tax revenues well. By so doing, it greatly narrows one of the elements of uncertainty in predicting tax revenues: model error. Accurate forecasting, however, also relies on correct values for the explanatory variables. Since these variables are not necessarily known as the forecast is being prepared, the quality of the projection obtained should be tested on the basis of the values of the macroeconomic variables as are known, or predicted, when the forecast is prepared.

As the first stage of this process, we need to profile the available data at the time the revenue forecast is constructed and test the quality of these variables against the actual data.

Table 7
Tax-Model Accuracy on the Basis of Actual Values of Explanatory Variables[1]

	Model Projection	Actual revenues	Absolute deviation	
Year	(NIS billions, 2000 prices)		(NIS billions, 2000 prices)	(Pct. of revenues)
2000	146.5	147.5	1.0	0.7
2001	148.9	148.5	0.4	0.3
2002	138.9	139.0	0.1	0.1
2003	133.2	134.3	1.1	0.8
2004	146.4	145.0	1.4	1.0
2005	155.3	155.9	0.6	0.4
2006	167.3	167.8	0.5	0.3
2000–2006 avg.			0.7	0.5

[1] The model projection for each year was calculated on the basis of the model's coefficients as estimated up to the end of the previous year.

a. Profiling the available data when the projection is being formulated

To test the forecasting quality of the tax model on the basis of data that are known before the tax year, we used retroactive estimates of the predicted variables for 1991–2006. For some variables, official forecasts were published when the state budget was prepared; we used these forecasts for these variables. For the other variables, we applied the rules that are used to produce the projection in practice.

GDP growth rate: the growth forecasts that we used were those appearing in the *National Budget*. The *National Budget*, co-published by the Bank of Israel and the Ministry of Finance every October until 2001, included a projection for the coming year. From 1992 on, it also included a two-year forecast. In some cases, the forecast comprised several scenarios; this happened in the early 1990s due to uncertainty about the extent of immigration. In these cases, we adopted the scenario that the *National Budget* defined as the reasonable one or the one based on the Bank of Israel's growth projection. In 2002 and subsequent years, after the *National Budget* was discontinued, the projections were based solely on the Bank of Israel's growth projection.[16] Since the growth forecast is expressed in annual terms, we assumed that the GDP growth rate was constant during the year.

Average wage per employee post and imports of consumption goods: these variables are included in the model as deviations from their long-term relation with GDP. The deviations were calculated as residuals derived from Engle-Granger cointegration equations between each of these variables and GDP at a one-quarter lag. In accordance with the error-correction coefficients that were found in these relations, the assumption in the forecast is that half of the wage deviation and all of the import deviation would be corrected in the following year.

Mergers and equity issues to parties of interest abroad: this variable displays strong irregularity and has no official estimates. Thus, the projection was calculated on the basis of actual data for the previous year divided equally among the quarters. To illustrate this, there were \$93 million in mergers and issues in the fourth quarter of 2005 after no mergers and issues whatsoever in the previous three quarters. Consequently, the mergers-and-issues projection for 2006 was \$23.5 million in each quarter.

Credit in or indexed to foreign currency: the projection for growth in foreign-currency credit was based on the spread between the Bank of Israel interest rate and the Fed rate, using the formula $g_t*[1+(\Delta r_t - 0.01)*10]$, where g is the expected GDP growth rate and Δr is the spread, in percentage points, between the effective Bank of Israel rate and the Fed rate. The baseline value for the calculation of change was the most recent known value when the forecast was being prepared.

New-dwelling sales: the projection was formulated on the assumption that sales increase in tandem with the pace of population growth at a two-year lag and the predicted rate of increase in **per-capita** GDP for that year.

[16] The difficulty in predicting GDP was especially conspicuous in 2003 and 2004, when no point forecast for the growth rate was presented; instead, two scenarios were offered (in each of the years). To perform the calculation, we used the average of the projections each year; this choice, however, obviously underestimates the uncertainty that the policymakers faced when they drew up the budget.

b. Quality of the explanatory variables' projections

To determine whether the predicted variables are reliable—i.e., whether they predict the actual values well—we calculated the forecasting error of the explanatory variables as a percentage of the values that actually turned out. To do this, we used estimates of the predictive variables for 1991–2006 as calculated in Section D1. Table 8 presents the averages and standard deviations of the predicted explanatory variables and the actual values. The two last columns in the table show the average absolute error in the sample period (1991–2006) and its rate as a percent of the average actual value of the variable. One may suppose that variables that exhibit large standard deviations are more difficult to predict. Indeed, values with relatively small variance, such as log GDP, appeared more conducive to accurate prediction during the sample period. In contrast, the variable of mergers and issues to parties of interest abroad showed very wide variance (it is presented in its original values and not in logs) because such transactions are not continuous.

Table 8
Accuracy of Predicted Explanatory Variables, 1991–2006 (NIS millions, in 2000 Prices)

Predicted variable	Projection		Actual values		Average absolute error	
	Avg.	S.D.	Avg.	S.D.	Avg.	S.D.
Real GDP	109,604	18,873	109,221	18,668	2,743	2.5
Wage	6,151	512	6,226	479	192	3.1
Imports	1,298	353	1,376	338	106	7.7
Mergers and issues	143	142	143	289	183	128.1
Foreign-currency credit	83,812	42,430	84,115	39,963	5,351	6.4
New-dwelling sales	8,685	2,115	8,565	1,641	1,958	22.9

The table shows that the average absolute error of the GDP projection was the smallest—2.5 percent. Thus, on average, each year's GDP growth rate was different from the budget forecast by 2.5 percentage points. This is a meaningful error from the standpoint of policymaking; furthermore, the forecast seems to suffer from a systematic bias related to the growth rate in the year when it was formulated (Appendix B). The prediction of mergers and issues to parties of interest abroad and new-dwelling sales suffered from especially large biases due to the irregularity of the data, which makes forecasting difficult.

The error in predicting wages was of a similar magnitude to that of predicting GDP, whereas the error in predicting imports of consumption goods was more than twice as large. Furthermore, relatively large deviations were observed in the error-correction estimates of these variables (i.e., correction of the deviation from the relation of the variables with GDP). These deviations had two origins: the effect of errors in the GDP projection—the correlation coefficient of the forecasting errors of wages with the forecasting errors of GDP was 0.4 in the sample period and that of imports was 0.15 (Appendix C)—and difficulty in predicting the exact relation of GDP with these variables.

5. ACCURACY OF THE MODEL'S PROJECTION AND TESTING OF ALTERNATIVE MODELS

In Table 9, we present the forecast that the model generated on the basis of estimates of the explanatory variables at the time the forecast was prepared as opposed to their actual values. The average absolute deviation of the revenue projection in 2001–2006 was NIS 4.5 billion—six times greater than the estimate based on actual data. This indicates that most of the forecasts' errors trace to difficulty in predicting the values of the explanatory variables and not in identification of the relation between the variables and revenues.

The quality of the point projection obtained from the tax model, as reflected in the distribution of the expected tax-revenue values, depends, as stated, both on the quality of the model itself and on the quality of the prediction of the explanatory variables and their covariance. Consequently, the model that provides the best *ex post* explanation of tax revenues is not necessarily the one that generates the best projections in terms of the scatter of the expected values, given the need to predict the values of the explanatory variables. In choosing the model that will generate the best forecast, we must admit that there is no single “true” model to which we can compare the projections. In other words, when we wish to choose a projection model, we need to minimize the information loss that originates in the difference between the available model and the unknown “true” model. To do this in our study, we used Akaike’s information criterion (1973) as phrased by Burnham and Anderson (2002) (hereinafter: AIC):

$$AIC = -2l + 2k \qquad (1)$$

Where k is the number of parameters estimated in the model, including the intercept, and l is the log likelihood of the model, given the historical data. Schwarz (1978) proposed an alternative criterion (hereinafter: SC) for the testing of model quality:

$$SC = -2l + k \cdot \log(n) \qquad (2)$$

Both criteria are based on twice the estimated likelihood and a component of loss. SC assigns greater weight than AIC to the insertion of additional explanatory variables.

Table 9
Accuracy of the Tax Model on the Basis of Predicted Values[1] (2000 prices)

Year	Actual revenues (NIS billions, 2000 prices)	Model forecast (NIS billions, 2000 prices)	Absolute deviation	(Pct. of revenues)
2001	148.5	155.9	7.4	5.0
2002	139.0	148.7	9.6	6.9
2003	134.3	136.8	2.5	1.9
2004	145.0	143.8	1.2	0.8
2005	155.9	154.1	1.8	1.1
2006	167.8	163.6	4.2	2.5
Avg.			4.5	3.0

[1] The model forecast for each year was calculated on the basis of the model's coefficients as estimated up to the end of the previous year.

To test the quality of the model on the basis of actual data, we first estimated cointegration including an intercept and log GDP as explanatory variables and took this model as a test case with which alternative models would be compared. In subsequent stages, alternative models including additional explanatory variables were estimated. Table 10 shows that according to both criteria, the addition of variables enhances explanatory power. The AIC criterion indicates that the best model is Model (4), which corresponds to the model estimated in Table 5 above. According to the SC criterion, however, Model (3) is better because the SC criterion "penalizes" adding explanatory variables that do not contribute enough to the model's explanatory power. By implication, mergers and issues abroad, new-dwelling sales, and the cost of foreign-currency credit do not contribute enough to the explanation of revenues to justify their inclusion in the forecast according to the SC criterion.

Table 10
Quality of the Model on the Basis of Actual Data[1]

	Model			
	1	2	3	4
Intercept	-1.77*	-2.11*	-2.99*	-4.17*
	(-.490)	(-.450)	(-.310)	(.714)
Log GDP (1-qtr. Lag)	1.05*	1.07*	1.15*	1.27*
	(-.040)	(-.040)	(-.030)	(.082)
Log GDP (1-qtr. Lag) * dummy variable for after-2000			-0.004*	-0.00*
			(.000)	(.001)
Change in log GDP (2-qtr. Lag)		1.18*	0.26	0.34*
		(-.300)	(-.160)	(.117)
Deviation of log wage from its relation with GDP			0.51*	0.53*
			(-.130)	(.116)
Deviation of log imports from its relation with GDP			0.54*	0.48*
			(-.050)	(.049)
Log imports deviation from its relation with GDP * dummy variable for after-2000			-0.17	-0.18*
			(-.090)	(.068)
Dummy for 2nd quarter			-0.01	-0.01**
			(-.010)	(.009)
Dummy for 4th quarter			-0.06*	-0.06*
			(-.010)	(.008)
Mergers and sales of shares abroad (2-qtr. Lag)				0.02**
				(.008)
Log foreign-currency credit (1-qtr. Lag)				-0.04**
				(.020)
Log new-dwelling sales (3-qtr. Lag)				0.02
				(.017)
Adjusted R-squared	0.91	0.93	0.99	0.99
Log likelihood	89	96	149	153
Akaike info criterion	-2.71	-2.91	-4.37	**-4.39**
Schwarz criterion	-2.64	-2.81	**-4.07**	-3.99

[1] Explained variable: total tax revenue net of effective legislative changes, in 2000 prices. Standard deviations are shown in parentheses. Model (4) is the same as that shown in Table 5.
* Significant at 1% level. ** Significant at 5% level.

In Table 11, we test the quality of the model estimated on the basis of data predicted at the time the forecast is prepared. Since the predicted explanatory variables are based on annual estimates interpolated with quarterly data, the model estimated on the basis of the predicted variables includes a component of first-order moving average (MA) in order to deal with structural errors in the forecasting model.[17] We find that the model preferred under SC in Table 10 is the best model according to both criteria (Equation 2). This is

Table 11
Choice of Model on the Basis of Estimation Using Predicted Data[1]

	Model					
	1	2	3	4	5	6
Intercept	-3.77* (-.870)	-3.12* (-.800)	-3.03* (-.820)	-3.83** (-1.520)	-3.33* (-.790)	-3.55** (-1.550)
Log GDP	1.23* (-.080)	1.17* (-.070)	1.18* (-.070)	1.25* (-.170)	1.19* (-.070)	1.23* (-.170)
Log GDP * dummy variable for after-2000	-0.01** (.000)	0.00** (.000)	0.00** (.000)	0.00** (.000)	-0.01** (.000)	-0.01** (.000)
Deviation of log wage from its relation with GDP		-0.36** (-.160)	-0.4** (-.170)	-0.37** (-.170)	-0.35** (-.170)	-0.39** (-.170)
Deviation of log imports from its relation with GDP		0.10 (-.070)	0.12 (-.070)	0.09 (-.070)	0.11 (-.070)	0.12 (-.080)
Dummy for 2nd quarter	-0.08* (-.010)	-0.06* (-.020)	-0.05** (-.020)	-0.06* (-.020)	-0.06* (-.020)	-0.05** (-.020)
Dummy for 3rd quarter	-0.05** (-.020)	-0.04** (-.020)	-0.03 (-.020)	-0.04** (-.020)	-0.04** (-.020)	-0.04** (-.020)
Dummy for 4th quarter	-0.09* (-.010)	-0.08* (-.020)	-0.08* (-.020)	-0.09* (-.020)	-0.08* (-.020)	-0.08* (-.020)
Mergers and sales of shares abroad					0.00 (.000)	0.00 (.000)
Log foreign currency credit				-0.02 (-.040)		-0.01 (-.040)
Log new-dwelling sales			-0.03 (-.030)			-0.03 (-.030)
Moving average of lagged residues of qtr.	0.42* (-.120)	0.36* (-.130)	0.39* (-.130)	0.33** (-.140)	0.32** (-.140)	0.34** (-.140)
Adjusted R-squared	0.95	0.96	0.96	0.96	0.96	0.96
Log likelihood	109	115	115	115	115	116
Akaike info criterion	-3.2	-3.36	-3.34	-3.33	-3.35	-3.3
Schwarz criterion	-2.96	-3.05	-3.00	-2.99	-3.01	-2.89

[1] Explained variable: total tax revenue net of effective legislative changes, in 2000 prices. Standard deviations are shown in parentheses.
* Significant at 1% level. ** Significant at 5% level.

[17] In Appendix D we examine—and rule out—the possibility that the model best suited to dealing with these errors is the ARMA structure.

because the three variables that were found to make only minor contributions to the model under SC are also the hardest to predict (Table 8). Therefore, the combination of the scanty contribution to explaining tax revenues and the difficulty in prediction excludes them from the most effective model for prediction. A comparison for 1992–2006 of the predictive quality of the equation chosen as against the predictive quality of the model shown in Table 6—on the basis of the predicted explanatory variables—shows that the forecast error of the model chosen is smaller: the average absolute deviation in these years was NIS 3 billion as against NIS 4.5 billion in the broader model. However, since the coefficients of this model are less stable, in a "rolling" estimate (Table 12) the accuracy of its projections is not much different from that of the broader model, for which the results of the estimation are shown in Table 9.

Table 12
Accuracy of Model Chosen on the Basis of Predicted Values of the Explanatory Variables1

	Actual revenues	Model forecast	Absolute deviation	
Year	(NIS billions, 2000 prices)		(Pct. of revenues)	
2001	148.5	153.7	5.2	3.5
2002	139.0	147.2	8.2	5.9
2003	134.3	139.2	4.9	3.6
2004	145.0	141.7	3.3	2.2
2005	155.9	155.7	0.2	0.2
2006	167.8	164.6	3.3	1.9
Avg.			4.2	2.9

[1] The model's projection for each year was calculated on the basis of the model coefficients as estimated up to the end of the previous year.

Due to the difficulty in predicting the explanatory variables, we tested for the possibility of obtaining a similar level of accuracy by using simpler forecasting models. To do this, we examined the accuracy of projections obtained from three conventional alternatives for projecting tax revenues: (1) econometric estimation of tax revenues in the next period on the basis of revenues in the current period, the growth rate, and the use of a moving average to solve the problem of serial correlation; (2) a method that presupposes unit elasticity of tax revenue, meaning that revenues increase in tandem with GDP; (3) a method assuming that the short-term elasticity of revenue to GDP is the same as the long-term elasticity (1.08); this is shown in Table 4. Comparison of the tax-revenue projection for 2000–2006, as derived from both methods, with that of the model chosen in Table 11 shows that the projection generated by the chosen model is much better than that produced by the alternative models (Table 13): the average absolute deviation of the alternative projections is NIS 5.4 billion in the sample period—4 percent of actual revenues—as against 2.4 percent in the chosen model. Using a "rolling" estimation of the model for 2001–2006, we found that it, too, points to the chosen model as superior (Table 14).

Table 13
Quality of 1991–2006 Tax-Revenue Forecast according to Different M Accuracy of Model Chosen on the Basis of Predicted Values of the Explanatory Variables1odels

	Average error		Absolute error	
Model	Avg.	Pct.	Avg.	Pct.
Model chosen on the basis of predicted data (Model 2 in Table 9)	-76	**0.0**	3,099	**2.4**
Lagged taxes and growth	4,611	3.4	5,291	4.0
Long-term elasticity (1.08)	-1,088	-0.9	5,352	4.1
Unit elasticity	-1,425	-1.1	5,439	4.1

Table 14
Absolute Deviation of Various Models (Percent of actual revenues)

Year	Limited model with predicted data	Lagged taxes and growth	Long-term elasticity (1.08)	Unit elasticity
2001	3.5	2.6	3.6	3.3
2002	5.9	4.7	9.0	8.9
2003	3.6	7.3	3.8	3.8
2004	2.2	5.2	5.8	5.9
2005	0.2	2.6	3.2	3.5
2006	1.9	1.8	2.1	2.4
Avg. error	2.9	4.1	4.6	4.6
In NIS billions	4.2	5.8	6.6	6.7

6. IMPLICIT UNCERTAINTY OF THE PROJECTION

Invoking the Bank of Israel's "inflation fan" methodology and the Bank of England's fan chart, we calculated the density function of the tax-revenue estimator on the basis of the model presented above. Having found that the tax model yields a very accurate description of the factors that affect tax revenue—when the explanatory variables are known—we calculated the uncertainty on the assumption that the empirical model is reliable and that the only source of error is in predicting the values of the explanatory variables.

There are two ways to derive the predicted variance of the explanatory variables and the predicted distribution of tax revenues. One of them uses a matrix of variances and covariances of the predicted variables relative to the explanatory variables—subjective judgment—as done by the Bank of England (Britton et al., 2006). In this method, the researcher establishes the values of some of the parameters at h/her discretion. Since the tax model uses annual forecasts for the explanatory variables and renders them into quarterly projections by linear transformation, this method will also lead by necessity to smaller variance of the predicted values. The second method, the one chosen in this study, is to derive the parameters by means of historical data and forecasting errors, thereby avoiding subjective judgment.

Unlike the inflation fan chart, since most of the predicted variables are based on annual estimates that are transformed into quarterly data, and since the forecast does not include lags of the dependent variable, the uncertainty remains constant across the year and does not rise from quarter to quarter.

Testing the quality of the projections of the variables included in the model that best describes the factors that affect tax revenues (Table 8), showed that most of the uncertainty originates in difficulty in predicting three explanatory variables: mergers and stock issues to parties of interest abroad, credit in, or indexed to, foreign currency, and new-dwelling sales. Testing the covariance of the variables, we also find that the covariance between the mergers-and-issues variable and the other explanatory variables in the model contributes to the uncertainty. In contrast, the contribution of GDP to errors in the projection is small, despite the large weight of GDP in explaining tax revenues. For this reason, we tested the probabilities of error in three alternative models: a simplistic model based on lagged revenues and the predicted growth rate, the comprehensive model shown in Table 6, and the model chosen in Table 11 (which excludes the three variables that are hard to predict).

To estimate the probability of tax revenues falling within a certain range, one needs to establish a forecast and make assumptions about the distribution of tax revenues and the extent to which the projection is symmetrically distributed. When the distribution is symmetric, its mean and mode are the same. When it is skewed, its mean and mode are not the same and the projection reflects the mode. In this case, the distribution is calculated by smooth-pasting two segments that belong to different normal distributions, satisfying two conditions: the sum of the probabilities must be 1 and the distribution around the mode must be skewed. In this method, the forecaster determines the extent of skewness *ab initio*. In our analysis, we assumed that the distribution is symmetric—a common assumption in both the monetary-fan chart of the Bank of Israel Research Department and in the Bank of England's fan chart.

The results of the estimation of the implicit risk in the tax-revenue forecast are shown in Table 15. The estimates for the three alternative models were calculated on the basis of predicted data for the explanatory variables in 2007 and a revenue-forecast distribution that was estimated on the basis of historical data. We found sizable differences among the models, especially in the probability of large errors. For example, the probability that the projection would be at least NIS 1.65 billion (0.25 percent of GDP) above actual revenues is 32 percent in the model that was found to be the most effective in Table 11 and the more broader model shown in Table 6, and 39 percent in the simplistic model. The larger the error we tested, the greater the disparity among the models became (Figure 2). In other words, adding variables beyond GDP helps to reduce the forecast error even if the variables are hard to predict. However, when we add to the model variables that are susceptible to especially poor predictability, the risk to the forecast increases even if they help to explain revenues.

Table 15
Probability of Tax-Revenue Errors according to Various Models

Deviation	Deviation	Chosen model	Original model	Unit elasticity
Pct. of GDP	(NIS millions)		Percent	
0.10	740	41.7	41.9	44.8
0.15	1000	38.8	39.1	43.0
0.25	1650	31.9	32.4	38.5
0.50	3300	17.4	18.0	28.0
1.00	6600	3.0	3.4	12.1

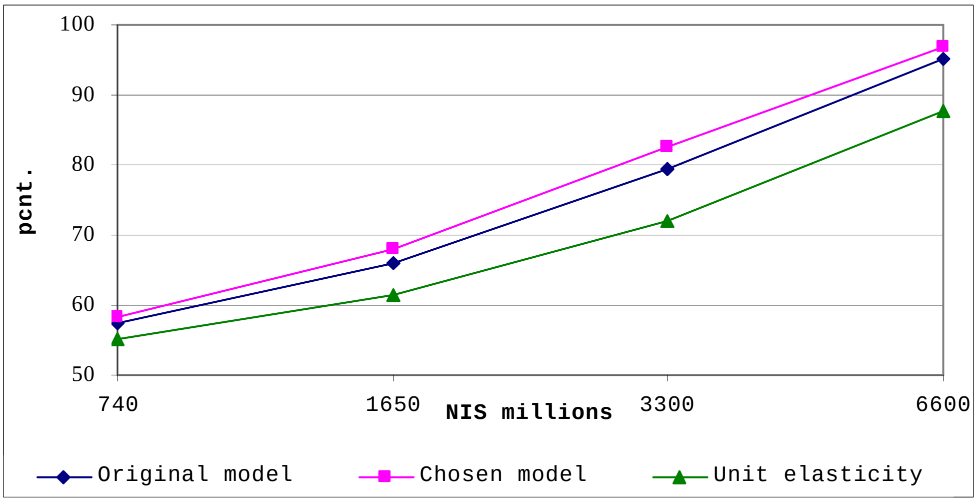
7. CONCLUSION

This study examined alternative models for the prediction of government tax revenues. For projection purposes, we divided the analysis into two stages. First we identified the real and financial macroeconomic variables that affect government tax revenues. We found that the model including GDP, imports of consumption goods, wage per employee post, new-dwelling sales, sales of shares by parties of interest abroad, and credit denominated in foreign currency successfully described the development of total revenues. The error in the model in annual terms was very small, it was stable over the years, and it explained revenues as well as separate models for individual taxes did. Furthermore, the macroeconomic variables excluded lagged tax revenues from the equation.

Although the model successfully **explains** the factors that affect tax revenues, the **forecasts** that may be generated by using it are not as good. This is because the values of the explanatory variables are unknown when the forecast is produced; therefore, the forecast can be based only on predicted values. We found that three variables—sales of shares by parties of interest abroad, credit denominated in foreign currency, and new-dwelling sales—impair the quality of the projections due to the difficulty in predicting their values effectively. For this reason—given the information that is available when the projection is produced—the projection as estimated on the basis of a limited model, which does not include these variables, has smaller errors than those based on the full model. By the same token, the original model is more stable and, therefore, in a “rolling” projection its errors are no larger than those of the limited version. The reason is that the errors in the predicted values of two other variables—foreign-currency credit and sale of shares abroad—correlate with the other variables in the model in a way that reduces the extent of the error in the revenue projection (Appendix C, Table C-1).[18]

[18] Since credit denominated in foreign currency reduces revenues, the positive correlation between the errors in the projection for this variable and the errors in the GDP forecast offsets some of the effects of the errors in the GDP forecast on the deviation of the revenue projection. The negative correlation between the errors of the projection for sales of shares by parties of interest to that of GDP has a similar moderating effect, especially since 2001.

Figure 2
Probability of Smaller Deviation of Revenues from Projection than the Number on the Horizontal Axis



Given the limits of information about the variables that explain tax revenues at the time the projection is prepared, we also examined two narrow models based only on lagged tax revenues and the expected GDP growth rate. The forecasts elicited by these models were inferior to those based on the full and limited models. In other words, despite the errors in predicting the explanatory variables, the forecasts based on their predicted values still bested those that totally disregard the relations between them and tax revenues. This superiority of the explanatory variables is also reflected in the probability of forecast error: the risk of errors in the narrow models is greater and the greater the potential error, the greater the difference between the narrow and broad models. Since policymakers seem to pay a higher “price” for large errors than for small ones, this makes the use of broader models advantageous.

This study points-out the main variables that affect government tax revenues. The difficulty in predicting some of the variables prompted us to use narrower models, thereby increasing the expected error in the revenue projection. It seems, then, that the course of action that offers the greatest marginal return in terms of the accuracy of the revenue projection in the budget is to improve the predictive ability of the explanatory variables. In the current study, we used simplistic estimates to predict some of the variables; a focused study might significantly improve the quality of the prediction—particularly in respect of the import and new-dwelling sales variables. Another proposed research task is the long-term testing of whether legislative changes relating to capital-market taxation will make financial variables more important in explaining revenues. An additional possible direction of research would be to ask whether some variables, although excluded from the model due to multicollinearity with other variables, may enhance predictive power because they are easier to predict than the included variables.

APPENDIX A

Table A-1
ADF Tests for Examination of Unit Root for Log Real GDP
(1973–2007, annual data)

Variable	Time trend	Statistical ADF	Significance
Log GDP	No	0.49	0.98
Log GDP	Yes	–1.78	0.69
Change in log GDP	No	–4.80*	0.00

Table A-2
ADF Tests for Exam+ination of Unit Root for Log Real GDP
(1990–2007, quarterly data)

Variable	Time trend	Statistical ADF	Significance
Log GDP	No	–1.20	0.67
Log GDP	Yes	–2.02	0.58
Change in log GDP	No	–12.59*	0.00

APPENDIX B: BIAS IN THE GDP PROJECTION

An important question in analyzing GDP projections, which figures so importantly in the estimation, is whether they are biased. To test for the presence of such a bias, we examined the real GDP forecasts for 1973–2006, as published in the *National Budget* for those years.[19] During this time, the real growth forecast to one year ahead was 3.5 percent on average while the actual growth rate was 4 percent per annum (Table B-1).[20] In other words, the projections were biased in a pessimistic direction, i.e., downward. From 1992 on, two-year projections were published as well. During this time, the average one year ahead forecast was 3.3 percent—0.7 percentage point under the actual GDP growth rate—and that for the subsequent year was 4.8 percent, slightly higher than actual GDP growth. Furthermore, the variance of the second-year forecast was lower than that of the first-year-ahead forecast, largely because the former was based on an estimate of potential product.

The downward bias in the growth projection for the coming year reflects forecasting errors in years when per-capita GDP growth was below the long-term average of 1.6 percent. In such years, the growth forecast for the coming year was more than 1.5 percentage point under the actual growth rate. In contrast, no bias was found in forecasts prepared in years of economic "boom" in and forecasts to two years ahead (in boom and bust years alike). The data also show that although the government has had to meet deficit targets since 1992, there is no meaningful difference between the average bias of the one-year projections in 1973–1991 and those in 1992–2006: in 1974–1993, the forecast to one year ahead was 0.6 percentage point below actual growth and in 1993–2006—after deficit targets were set—it was 0.7 percentage point below.

The cumulative deviation of the projection to two years ahead was only 0.2 percentage point. Nevertheless, despite the average accuracy, the variance of the error in the GDP forecast to two years ahead was twice as great as that to one year ahead—because the error in the one-year forecast was carried into the second year as well. Furthermore, the average absolute error of the cumulative two-year forecast during this time was 3.3 percentage points as against an average error of 1.7 percentage point to one year ahead. Consequently, the uncertainty to two years ahead was no less than that to one year ahead, due to the risk carried forward in the projection from year to year.

[19] In the years after the termination of the *National Budget* we used the projections published of the Bank of Israel in parallel to the budget's submission to the Knesset.

[20] Geometric average of the real growth rate in 1973–2006. During this time, the population grew by 2.3 percent per annum and real per-capita growth was 1.6 percent per annum.

Table B-1
Accuracy of Real GDP Projection to One Year and Two Years Ahead, 1973–2006[1]

Year	Projection: To 1 year ahead	Projection: Cumulative to 2 years ahead	Actual real growth	Forecast error: To 1 year ahead	Forecast error: Cumulative to 2 years ahead
1973	8.8	..	4.9	3.9	..
1974	6.9	..	5.5	1.4	..
1975	3.2	..	3.8	-0.6	..
1976	1.0	..	1.6	-0.6	..
1977	1.2	..	2.0	-0.8	..
1978	4.0	..	4.1	-0.1	..
1979	6.0	..	4.7	1.3	..
1980	4.2	..	3.6	0.6	..
1981	2.7	..	4.7	-2.0	..
1982	0.0	..	1.4	-1.4	..
1983	1.8	..	2.6	-0.8	..
1984	-1.3	..	2.2	-3.5	..
1985	0.8	..	4.5	-3.7	..
1986	1.6	..	3.6	-2.0	..
1987	3.2	..	6.2	-3.0	..
1988	3.7	..	3.6	0.1	..
1989	1.9	..	1.4	0.5	..
1990	5.0	..	6.6	-1.6	..
1991	7.1	..	6.1	1.0	..
1992	6.5	..	7.1	-0.6	..
1993	3.6	13.2	3.8	-0.2	2.0
1994	5.3	9.9	7.0	-1.7	-1.1
1995	4.9	11.5	6.6	-1.7	-2.5
1996	5.0	10.0	5.6	-0.6	-2.5
1997	4.0	10.1	2.8	1.2	1.6
1998	3.1	9.3	4.2	-1.1	2.2
1999	2.5	8.0	2.9	-0.4	0.8
2000	3.0	6.6	8.7	-5.7	-5.3
2001	4.0	7.9	-0.6	4.6	-0.1
2002	1.9	9.2	-0.9	2.8	10.8
2003	0.3	6.3	1.5	-1.3	5.7
2004	1.6	..	4.8	-3.2	..
2005	3.8	4.4	5.2	-1.4	-5.9
2006	3.9	7.7	5.1	-1.2	-2.8
Avg.	**3.5**	**8.8**	**4.0**	**-0.6**	**0.2**
Variance	**4.7**	**5.4**	**4.8**	**4.3**	**20.4**

[1] The averages in the cumulative two-year forecast are for 1993–2006.

APPENDIX C
Table C-1
Matrix of Correlation Coefficients of Errors in Predicted Explanatory Variables (Q1:1991–Q4:2006)

	Log lagged GDP	Log lagged GDP*, dummy variable for after 2000	Deviation of log wage from relation with GDP	Deviation of log consump-tion-goods imports from relation with GDP	Deviation of log consump-tion-goods imports * dummy variable for after 2000	Mergers and sales of shares abroad At 2-qtr. lag	Log forex credit at 1-qtr. lag	Change in growth rate at 2-qtr. lag	Log new-dwelling sales at 3-qtr. lag
Log lagged GDP	1	0.34	0.24	-0.10	-0.20	-0.05	-0.05	-0.15	0.09
Log lagged GDP*, dummy variable for after 2000		1	0.37	-0.20	-0.23	-0.29	-0.29	-0.02	-0.01
Deviation of log wage from relation with GDP			1	0.07	0.32	-0.18	-0.18	0.08	0.15
Deviation of log consumption-goods imports * dummy variable for after 2000				1	0.65	0.18	0.18	0.07	-0.13
Mergers and sales of shares abroad at 2-qtr. lag					1	0.11	0.11	-0.04	0.15
Log forex credit at 1-qtr. lag						1	1	-0.26	0.02
Change in growth rate at 2-qtr. lag								0.02	-0.03
Log new-dwelling sales at 3-qtr. lag								1	-0.18

APPENDIX D: TESTING THE STRUCTURE OF THE RESIDUALS

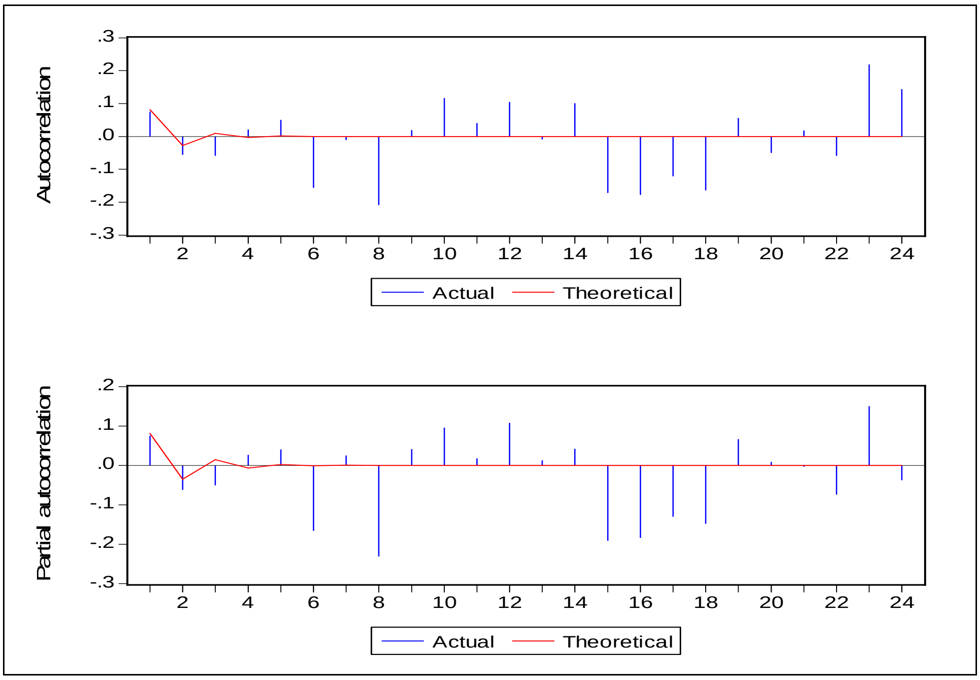
To determine whether the forecasting model is stationary and whether an autoregressive process that entails the use of FGLS is present, we performed three tests for ARMA structure. The tests show that a stationary process does exist. This is because the model explains tax revenues by using exogenous variables and excludes lagged taxes.

First, we conducted a test to determine the root of the residuals. The test shows that both the AR and the MA are within unit root and that, therefore, the residual is stationary; furthermore, they are strongly correlated (Table D-1). Second, we examined the correlogram of the residues (Figure D-1) and found no evidence of an autoregressive process. In other words, the finding corresponds to our model, in which tax revenues are explained by GDP and not by lagged taxes. Finally, we performed an impulse-response test; it showed that the process converges to zero rapidly—within two quarters—indicating both that the residuals are stationary and that they do not exhibit an autoregressive process.

Table D-1
Root Test

Root	Model ARMA(1,1)	Model MA(1)
MA(1)	–0.43157	–0.10022
AR(1)	–0.346946	

Figure D-1: Correlogram



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